SLOW WAVES IN A POROELASTIC SOLID SATURATED BY MULTIPHASE FLUIDS

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Introduction. I

- The theory of propagation of waves in a poroelastic solid saturated by a single-phase fluid was presented by M. Biot.
- The generalization to the case of multiphase saturating fluids requires taking into account capillary pressures and relative permeability functions.
- To obtain the constitutive relations we apply the principle of virtual complementary work, where capillary forces are included as restrictions using Lagrange multipliers.
- To determine the elastic constants in the constitutive relations in terms of the properties of all phases, we apply a set of ¨gedanken¨ experiments.

Introduction. II

- Multiphase Darcy law including relative permeabilities are used in the definition of the dissipation functions, leading to the Lagrangian formulation of the equations of motion.

- Wave induced fluid flow and slow waves are a major cause of attenuation and velocity dispersion of seismic waves in Biot media, which occur at the mesoscopic scale, on the order of centimeters, but their effect can be observed at the macro-scale

Objectives

- We generalize Biot´s theory to the case of multiphase fluids including capillary pressure and relative permeability functions.
- For two-phase fluids, a plane wave analysis shows the existence of three compressional waves (P1, P2, P3) and one shear (S) wave.
- For three-phase fluids, an additional compressional P4 wave exists.
- While the P1, P2 and S waves behave similar to those in the classical Biot theory, the P3 and P4 compressional waves are additional slow waves whose behavior is ruled by the motion of the non-wetting phases.

Outline

- **Theory**

Constitutive relations: two-phase immiscible fluids
Constitutive relations: three-phase immiscible fluids
Equations of motion: three-phase fluid case
Plane Wave Analysis: three-phase fluid case for compressional waves

- **Numerical Examples**

- **Conclusions**

Theory

Constitutive relations: Two-phase fluids

Ω : poroelastic isotropic medium saturated by a two-phase fluid

$$\tau_{ij} = 2\,G\,\varepsilon_{ij} + \delta_{ij}(\lambda_c\,\nabla\cdot u^s + B_1\,\nabla\cdot u^{nw} + B_2\,\nabla\cdot u^w)$$

$$F^{nw} = -B_1\nabla\cdot u^s - M_1\nabla\cdot u^{nw} - M_3\nabla\cdot u^w$$

$$F^{w} = -B_2\nabla\cdot u^s - M_3\nabla\cdot u^{nw} - M_2\nabla\cdot u^w$$

τ_{ij} : total stress tensor $\qquad \varepsilon_{ij}\left(u^s\right)$: strain tensor

$u^s, \tilde{u}^\theta$: solid and θ -fluid displacements averaged over the bulk material

$u^\theta = \phi\left(\tilde{u}^\theta - u^s\right)$, ϕ : porosity $\qquad \theta = w,\ nw$

G : shear modulus of the dry matrix $\qquad \lambda_c = K_u - \frac{2}{3}G$, K_u : undrained bulk modulus

Constitutive relations: Two-phase fluids

$F^{nw}, \; F^{w}$: generalized fluid forces

$$F^{w} = (S_w + \zeta)\, p_w - \zeta\, p_{nw} \quad , \qquad \zeta = \bar{p}_w / Pc'\,(S_{nw})$$

$$F^{nw} = (S_{nw} + \beta + \zeta)\, p_{nw} - (\beta + \zeta)\, p_w \,, \quad \beta = Pc\,(S_{nw}) / Pc'(S_{nw})$$

S_{θ} : θ-fluid saturation, $S_w + S_{nw} = 1$ $\qquad p_{\theta}$: θ-fluid pressure

$Pc(S_{nw}) = p_{nw} - p_w$: capillary pressure $\qquad \bar{p}_w$: reference wetting fluid pressure

The elastic constants M_1, M_2, M_3, B_1, B_2 are determined applying a set of "gedanken experiments", generalizing those of the single fluid phase of Biot

Constitutive relations: Three-phase fluids

Ω : poroelastic isotropic homogeneous medium saturated by oil (o), water (w) and gas (g)

$$\tau_{ij} = 2\,G\,\varepsilon_{ij} + \delta_{ij}(\lambda_c\,\nabla\cdot u^s + B_1\,\nabla\cdot u^o + B_2\,\nabla\cdot u^w + B_3\,\nabla\cdot u^g)$$

$$F^o = -B_1\,\nabla\cdot u^s - M_1\,\nabla\cdot u^o + M_4\,\nabla\cdot u^w + M_5\,\nabla\cdot u^g$$

$$F^w = -B_2\,\nabla\cdot u^s - M_4\,\nabla\cdot u^o + M_2\,\nabla\cdot u^w + M_6\,\nabla\cdot u^g$$

$$F^g = -B_3\,\nabla\cdot u^s - M_5\,\nabla\cdot u^o + M_6\,\nabla\cdot u^w + M_3\,\nabla\cdot u^g$$

S_o , S_w , S_g : oil, water and gas saturations, $S_o + S_w + S_g = 1$

p_o , p_w , p_g : oil, water and gas pressures

$$Pc_{ow}(S_o) = p_o - p_w$$

$$Pc_{go}(S_g) = p_g - p_o$$

Constitutive relations: Three-phase fluids

$F^o,\ F^w,\ F^g$: generalized fluid forces

$$F^o \equiv (S_o + \beta_{ow} + \beta_{ow}^w)p_o - (\beta_{ow} + \beta_{ow}^w)p_w$$

$$F^w \equiv (S_w + \beta_{ow}^w)p_w + \left(\beta_{go}^w - \beta_{ow}^w\right)p_o - \beta_{go}^w p_g$$

$$F^g \equiv \left(S_g + \beta_{go} + \beta_{go}^{ow} + \beta_{go}^w\right)p_g - \left(\beta_{go} + \beta_{go}^{ow} + \beta_{go}^w\right)p_o$$

$$\beta_{ow} = \frac{Pc_{ow}(S_o)}{Pc'_{ow}(S_o)} \qquad \beta_{ow}^w = \frac{\bar{p}_w}{Pc'_{ow}(S_o)} \qquad \beta_{go}^w = \frac{\bar{p}_w}{Pc'_{go}(S_g)}$$

$$\beta_{go} = \frac{Pc_{go}(S_g)}{Pc'_{go}(S_g)} \qquad \beta_{go}^{ow} = \frac{Pc_{ow}(S_o)}{Pc'_{go}(S_g)}$$

The elastic constants M_j, B_j, are determined applying a set of “gedanken experiments”.

Equations of motion: three-phase fluid

Assuming constant coefficients and absence of external sources :

$$\rho \ddot{u}^s + \rho_o S_o \ddot{u}^o + \rho_w S_w \ddot{u}^w + \rho_g S_g \ddot{u}^g = \nabla \cdot \tau(\vec{u})$$

$$\rho_o S_o \ddot{u}^s + g_o \ddot{u}^o + g_{ow} \ddot{u}^w + g_{og} \ddot{u}^g + b_o \dot{u}^o + b_{ow} \dot{u}^w + b_{og} \dot{u}^g = -\nabla F_o(u)$$

$$\rho_w S_w \ddot{u}^s + g_{ow} \ddot{u}^o + g_w \ddot{u}^w + g_{wg} \ddot{u}^g + b_{ow} \dot{u}^o + b_w \dot{u}^w + b_{wg} \dot{u}^g = -\nabla F_w(u)$$

$$\rho_g S_g \ddot{u}^s + g_{og} \ddot{u}^o + g_{wg} \ddot{u}^w + g_g \ddot{u}^g + b_{og} \dot{u}^o + b_{wg} \dot{u}^w + b_g \dot{u}^g = -\nabla F_g(u)$$

Mass and viscous coupling coefficients:

$$g_\theta = \frac{S_\theta \, \rho_\theta T}{\phi} \qquad g_{st} = \varepsilon\left(g_o \, g_w \, g_g\right)^{\frac{1}{3}} \qquad b_\theta = \frac{S_\theta^2 \, \eta_\theta}{k \;\; k_{r\theta}} \qquad b_{st} = \varepsilon\left(\frac{S_o^2}{k_{ro}} \frac{S_w^2}{k_{rw}} \frac{S_g^2}{k_{rg}}\right)^{\frac{1}{3}} \frac{(\eta_s \, \eta_t)^{\frac{1}{2}}}{k}$$

ρ_θ: density T: tortuosity η_θ : viscosity k, $k_{r\theta}$: absolute and relative permeabilities

Equations of motion: three phase fluid

In compact form:

$$P\left(\ddot{u}_i^s, \ddot{u}_i^o, \ddot{u}_i^w, \ddot{u}_i^g\right)^t + D\left(\dot{u}_i^s, \dot{u}_i^o, \dot{u}_i^w, \dot{u}_i^g\right)^t$$

$$= \left(\frac{\partial \tau_{ij}}{\partial x_j}, \frac{\partial F^o}{\partial x_i}, \frac{\partial F^w}{\partial x_i}, \frac{\partial F^g}{\partial x_i}\right)^t \quad , \quad i=1,2,3$$

P : mass coupling 12x12 symmetric positive definite matrix, defined in terms of the mass coefficients of the individual phases

D : viscous coupling 12x12 symmetric non-negative matrix, defined in terms of the viscosity of the individual fluid phases, the absolute permeability and the three phase relative permeabilities.

Plane Wave Analysis: compressional waves

The divergence operator is applied to the equations of motion. Besides a plane compressional wave of angular frequency ω and wave number $l = l_r + i\, l_l$ travelling in the x_1-direction is considered:

$$e^{\theta} = \nabla \cdot u^{\theta} = C_{\theta} e^{i\,(l\,x_1 - \omega t\,)} \quad , \qquad \theta = s, o, w, g$$

Replacing in the motion equations and using the expression of the generalized forces:

$$\begin{aligned}
&-\omega^2 \rho e^s - \omega^2 \rho_o S_o e^o - \omega^2 \rho_w S_w e^w - \omega^2 \rho_g S_g e^g \\
&\qquad = (\lambda_c + 2\mathrm{G})\, \nabla^2 e^s + B_1 \nabla^2 e^o + B_2 \nabla^2 e^w + B_3 \nabla^2 e^g
\end{aligned}$$

$$\begin{aligned}
&-\omega^2 \rho_o S_o e^s - \omega^2 g_o e^o - \omega^2 g_{ow}\, e^w - \omega^2 g_{og}\, e^g + i\omega b_o e^o + i\omega b_{ow} e^w + i\omega b_{og} e^g \\
&\qquad = B_1 \nabla^2 e^s + M_1 \nabla^2 e^o + M_4 \nabla^2 e^w + M_5 \nabla^2 e^g
\end{aligned}$$

$$\begin{aligned}
&-\omega^2 \rho_w S_w e^s - \omega^2 g_{ow} e^o - \omega^2 g_w\, e^w - \omega^2 g_{wg}\, e^g + i\omega b_{ow} e^o + i\omega b_w e^w + i\omega b_{wg} e^g \\
&\qquad = B_2 \nabla^2 e^s + M_4 \nabla^2 e^o + M_2 \nabla e^w + M_6 \nabla^2 e^g
\end{aligned}$$

$$\begin{aligned}
&-\omega^2 \rho_g S_g e^s - \omega^2 g_{og} e^o - \omega^2 g_{wg}\, e^w - \omega^2 g_g\, e^g + i\omega b_{og} e^o + i\omega b_{wg} e^w + i\omega b_g e^g \\
&\qquad = B_3 \nabla^2 e^s + M_5 \nabla^2 e^o + M_6 \nabla^2 e^w + M_3 \nabla^2 e^g
\end{aligned}$$

Plane Wave Analysis: compressional waves

Setting $\gamma = \dfrac{\omega}{l}$ the following eigenvalue problem is obtained:

$$\gamma^2 A\, C^l = E\, C^l$$

$$A = \begin{pmatrix} \rho & \rho_o S_o & \rho_w S_w & \rho_g S_g \\ \rho_o S_o & g_o + i\frac{b_o}{\omega} & g_{ow} + i\frac{b_{ow}}{\omega} & g_{og} + i\frac{b_{og}}{\omega} \\ \rho_w S_w & g_{ow} + i\frac{b_{ow}}{\omega} & g_w + i\frac{b_w}{\omega} & g_{wg} + i\frac{b_{wg}}{\omega} \\ \rho_g S_g & g_{og} + i\frac{b_{og}}{\omega} & g_{wg} + i\frac{b_{wg}}{\omega} & g_g + i\frac{b_g}{\omega} \end{pmatrix} \qquad E = \begin{pmatrix} \lambda_c + 2N & B_1 & B_2 & B_3 \\ B_1 & M_1 & M_4 & M_5 \\ B_2 & M_4 & M_2 & M_6 \\ B_3 & M_5 & M_6 & M_3 \end{pmatrix}$$

To determine $l = l_r + i\, l_l$ we solve $det(A^{-1}E - \gamma^2 I) = 0$

The four physical meaningful solutions determine the phase velocities v_{pj} and attenuation coefficients a_{pj} of the P1, P2, P3 and P4 compressional waves

$$v_{pj} = \frac{\omega}{|l_{rj}|} \qquad a_{pj} = 2\pi\,.\,8.655588\,\frac{|l_{ij}|}{|l_{rj}|} \qquad j = 1, 2, 3, 4$$

Numerical Examples

We analyze the behavior of all slow waves as functions of saturation and frequency for different $\bar{p}_w$ in a sample of Berea Sandstone.

$\phi = 0.3$; $k = 1.0$ D; $\rho_s = 2.65$ gr/cm^3, $K_s = 37$ GPa

FLUID PROPERTIES	OIL	WATER	GAS		
			pw=10 MPa	pw=30 MPa	pw=50 MPa
ρ (kg/m^3)	0.7	1	86.52	185.84	218.29
η (Pa.s)	0.1	0.01	$1.17\ 10^{-5}$	$1.39\ 10^{-5}$	$1.59\ 10^{-5}$
K (GPa)	0.57	2.25	$8.93\ 10^{-3}$	$4.08\ 10^{-2}$	$7.98\ 10^{-2}$
Matrix Properties		K_m (GPa)	12.79	15.57	15.62
		G (GPa)	11.81	13.61	13.70

Capillary pressure functions

$$Pc_{ow}(S_o) = A_{ow}\left(1/(S_o + S_{rw} - 1)^2 - S_{ro}^2/[S_o(1 - S_{ro} - S_{rw})]^2\right), \quad S_{ro} \le S_o \le 1 - S_{rw} - S_{rg}$$

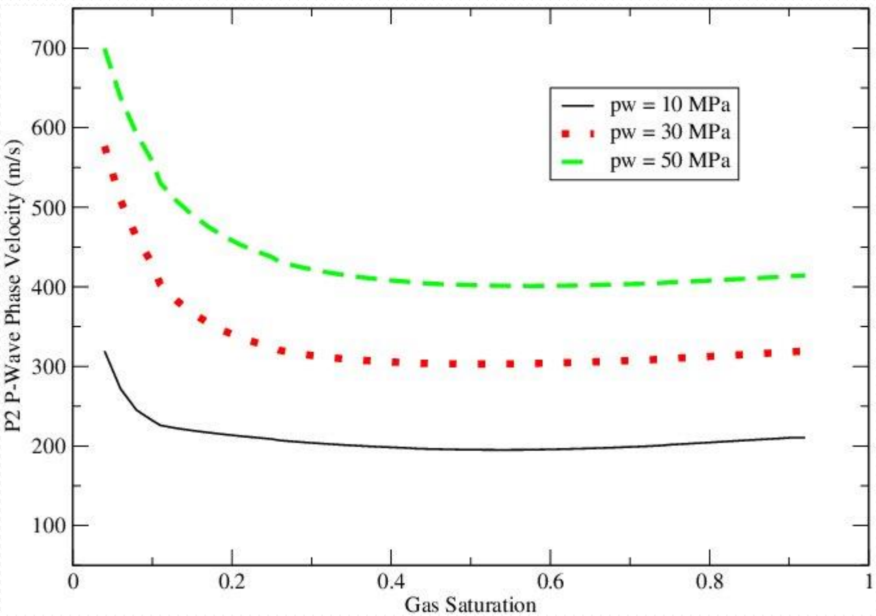
$$Pc_{go}(S_g) = A_{go}\left(1/(S_g + S_{rw} - 1)^2 - S_{rg}^2/[S_g(1 - S_{rg} - S_{rw})]^2\right), \quad S_{ro} \le S_o \le 1 - S_{rw} - S_{rg}$$

Relative permeability functions

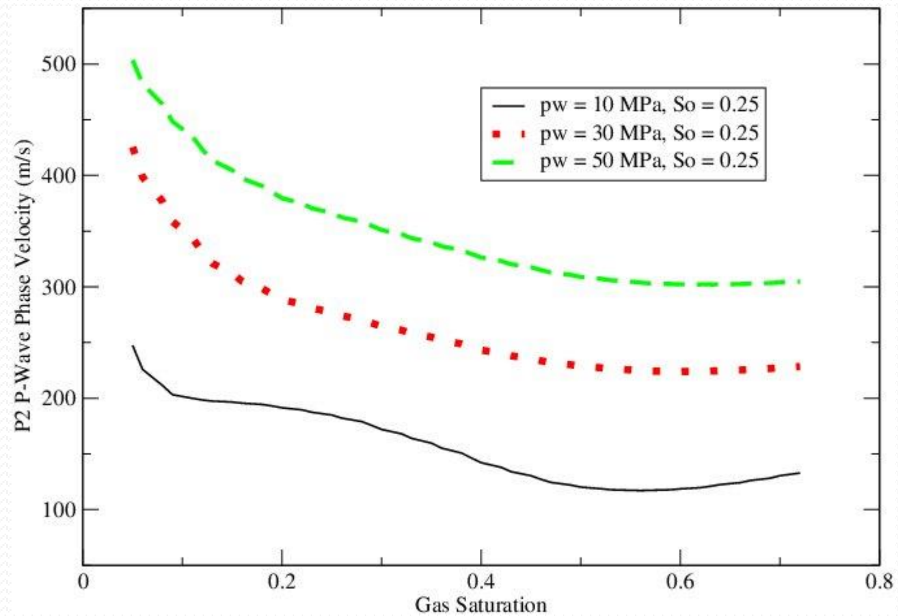
$$k_{r\theta}(S_\theta) = \left(\frac{S_\theta - S_{r\theta}}{1 - S_{r\theta}}\right)^2, \quad S_{r\theta} \le S_\theta \le 1 - S_{rp} - S_{rq}, \; p \ne \theta, q \ne \theta, \; p \ne q$$

P2 P-Wave Phase Velocity

Two-phase fluid



Three-phase fluid

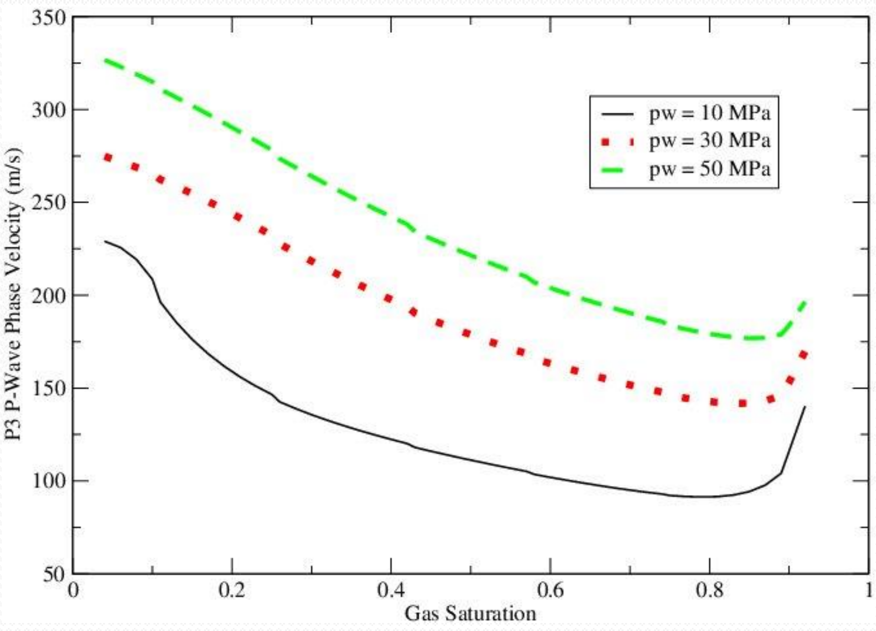


P2 wave velocities display a decreasing behavior at low gas saturation and stabilize at higher saturations.
Velocity increases for increasing values of water pressure.

P2 wave phase velocities have a general decreasing behavior as gas saturation increases.
Velocity increases for increasing values of water pressure.

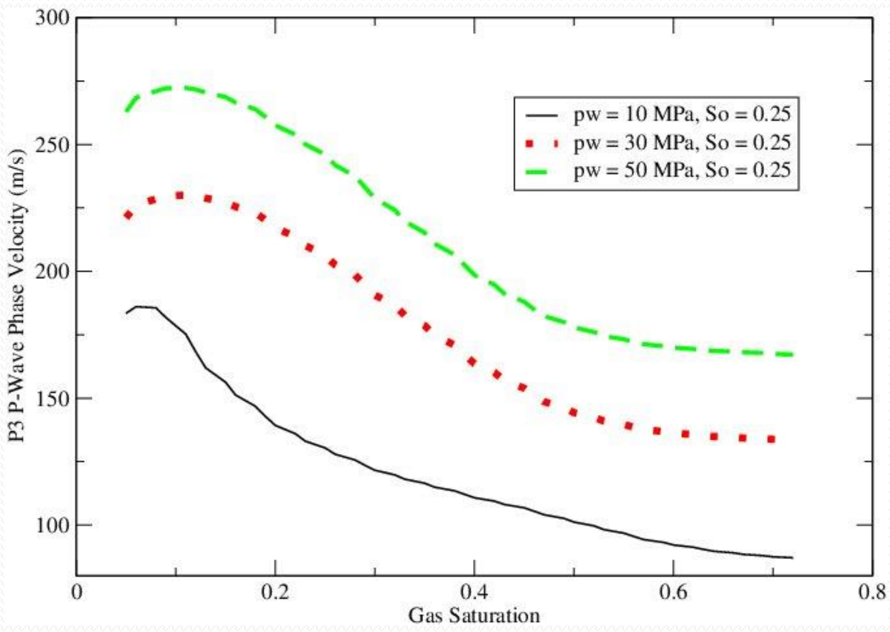
P3 P-Wave Phase Velocity

Two-phase fluid



P3 wave velocities show a general decreasing behavior as gas saturation increases, except in high saturations, where they start to increase.

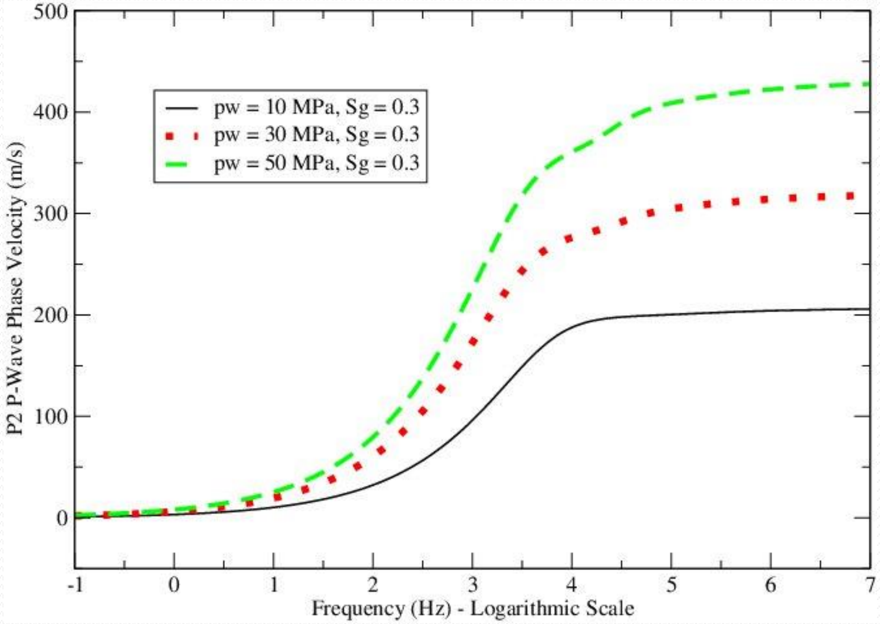
Three-phase fluid



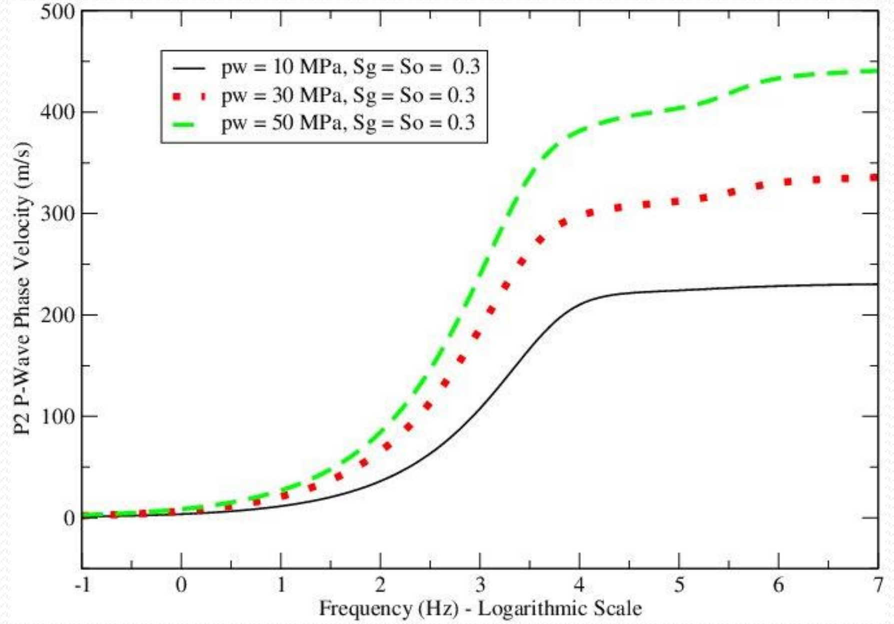
P3 wave velocities show a general decreasing behavior as gas saturation increases, except in low saturations, where they increase.

P2 P-Wave Phase Velocity

Two-phase fluid



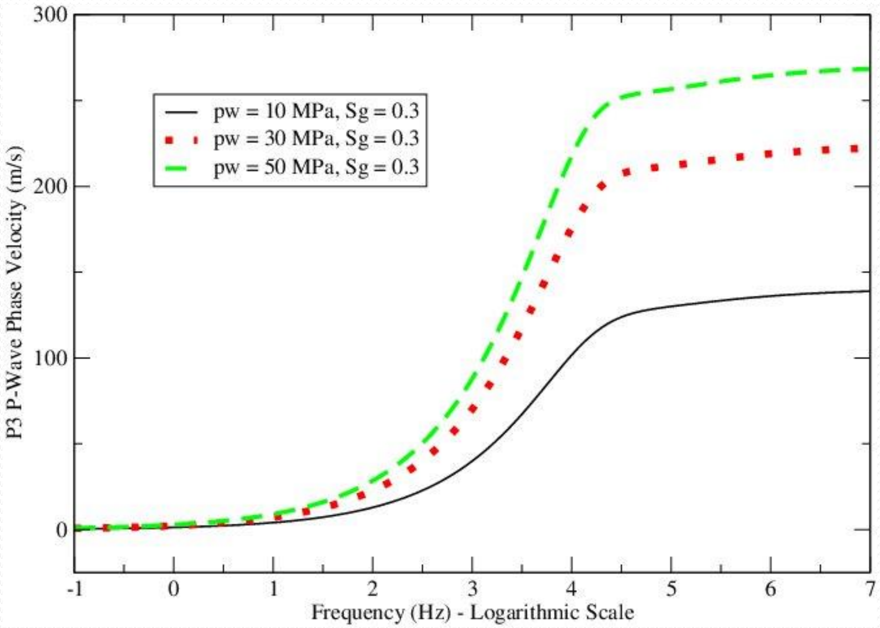
Three-phase fluid



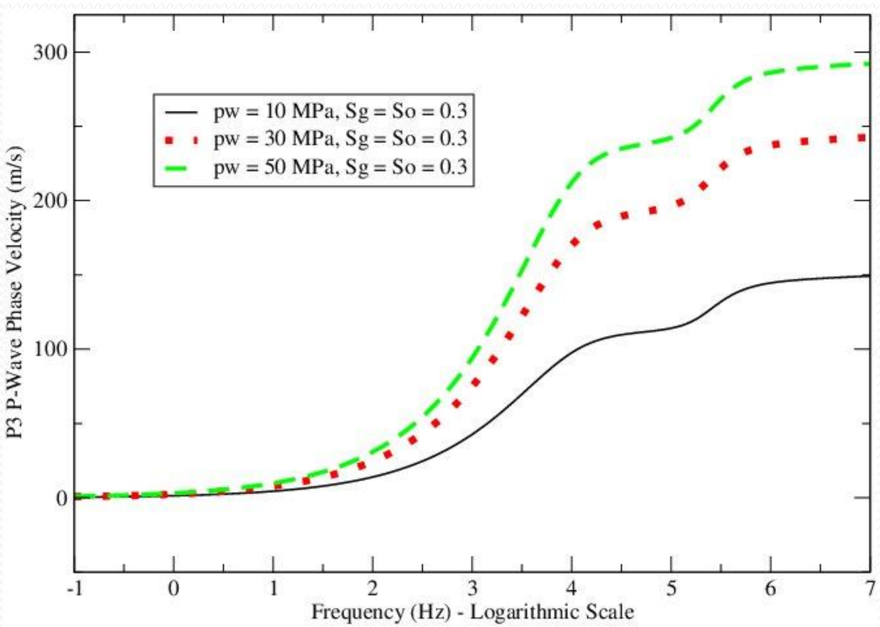
P2 wave velocities are almost zero at low frequencies, behaving as diffusion-type waves, while they stabilize at high frequencies. Furthermore, they increase as the water reference pressure increases.

P3 P-Wave Phase Velocity

Two-phase fluid

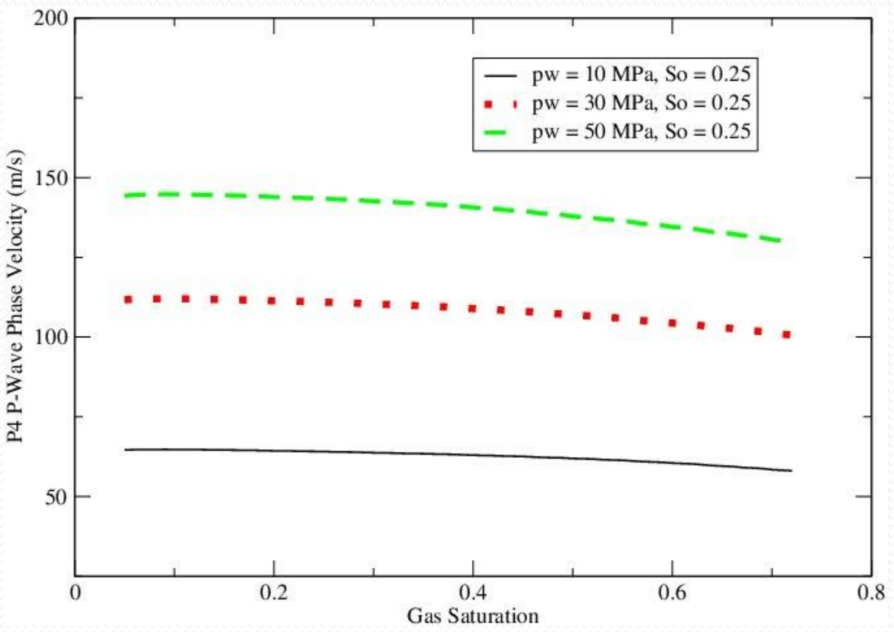


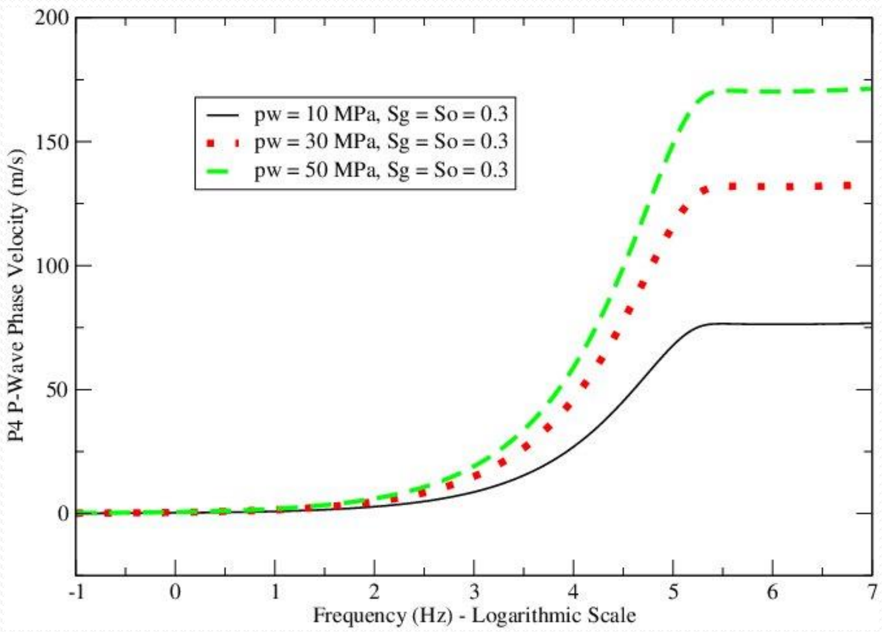
Three-phase fluid



P3 wave velocities are almost zero at low frequencies, behaving as diffusion-type waves, while they stabilize at high frequencies. Furthermore, they increase as the water reference pressure increases.

P4 P-Wave Phase Velocity



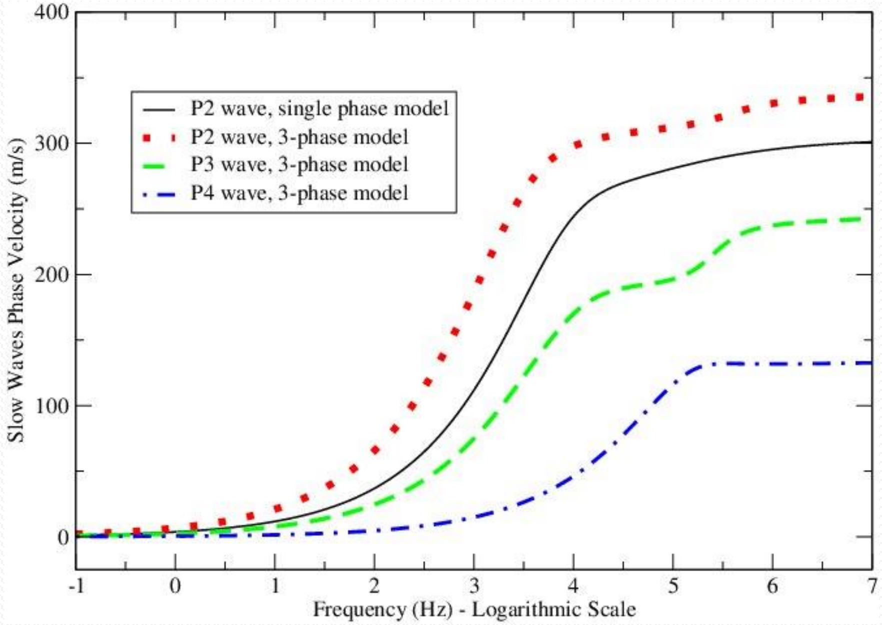


P4 wave velocities are almost independent of gas saturation with their values increasing as water pressure increases.

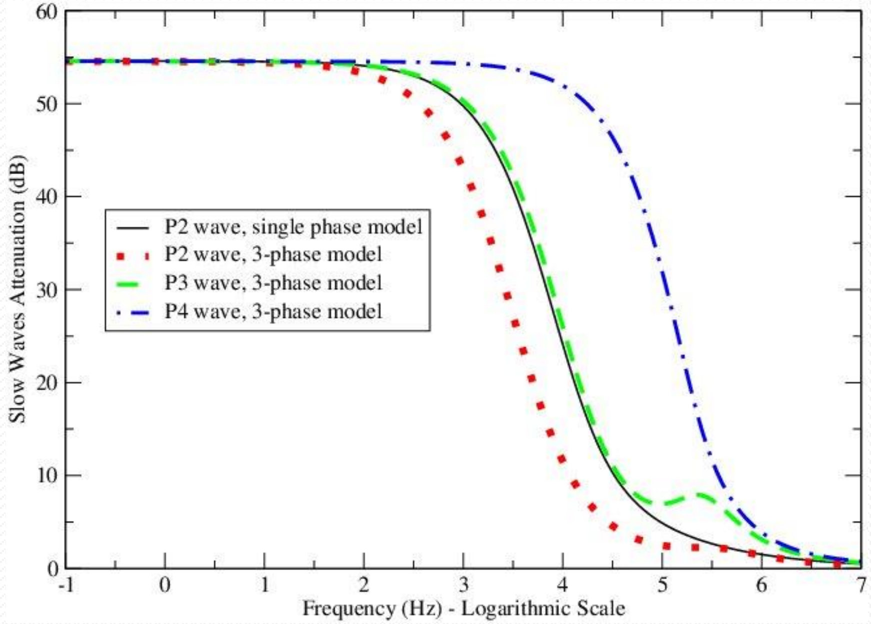
P4 P-wave as function of frequency behaves similarly as P2 and P3 P-waves. Furthermore, velocities increase as water reference pressure increases

Comparison between the classic Biot theory and the three-phase model

Velocity



Attenuation



The classic P2 P-wave phase velocity (left) and attenuation (rigth) have intermediate values among the slow waves of the three-phase model

Conclusions

- This work presents an analysis of the behavior of slow waves in a poroelastic solid saturated by multiphase fluids.
- At 1 MHz, slow P2 and P3 P-waves phase velocities increase as function of depth and show a general decreasing behavior as function of the non-wetting saturation.
- Slow P2 and P3 P-waves are diffusion type waves at low frequencies and stabilize at the ultrasonic range; they exhibit higher values with increasing pressure.
- At 1 MHz, slow P4 waves are almost independent of gas saturation and, as function of frequency, behave as the slow P2 and P3 waves

Conclusions

- A comparison among velocity and attenuation of the classic P2 Biot wave and the slow waves of the three-phase model shows that all of them have a similar behavior, and, for the chosen saturation values, the classic P2 wave takes intermediate velocity and attenuation values among those of the three-phase model

Thank you!