Analysis of Fracture Induced Anisotropy in a Biot Medium as Function of Effective Pressure

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Summary

The acoustic properties of fractured hydrocarbon reservoirs vary with formation depth and pore pressure, with seismic velocity and attenuation being function of the effective pressure, the difference between confining and pore pressure. This work analyzes variations in velocity and attenuation in a Biot medium containing a dense set of aligned fractures due to changes in pore pressures. Fractures are modeled as boundary conditions imposing discontinuity of pore pressures and solid and fluid displacements along fractures. Using numerical experiments and for wavelengths much larger than the average fracture distance and aperture, it is possibly to determine a viscoelastic transversely isotropic (VTI) medium long-wave equivalent to the original fractured Biot medium. It is shown that velocity and attenuation of the equivalent VTI medium are noticeable sensitive to variations in pore pressure.

A fractured Biot medium

We consider a fractured Biot medium and assume that the whole aggregate is isotropic. Let the super index $\theta = b, f$, indicate solid matrix and saturant fluid properties associated with the background and fractures, respectively.

Let $\mathbf{u}_s = (u_{s,i})$ and $\tilde{\mathbf{u}}_f = (\tilde{u}_{f,i})$, $i = 1,\cdots,3$, denote the averaged displacement vectors of the solid and fluid phases, respectively. Also let

$$\mathbf{u}_f = \phi^{(\theta)}(\tilde{\mathbf{u}}_f - \mathbf{u}_s), \quad \xi = -\nabla \cdot \mathbf{u}_f.$$

Set $\mathbf{u} = (\mathbf{u}_s,\ \mathbf{u}_f)$ and let $\mathbf{e}(\mathbf{u}_s) = e_{st}(\mathbf{u}_s)$ be the strain tensor of the solid. Also, let $\boldsymbol{\sigma}(\mathbf{u})$ and $p_f(\mathbf{u})$ denote the stress tensor of the bulk material and the fluid pressure, respectively.

The linear stress-strain relations in a fractured Biot medium are (Biot, 1962):

$$\sigma_{st}(\mathbf{u}) = 2G^{(\theta)} e_{st}(\mathbf{u}_s) + \delta_{st}\left(\lambda_U^{(\theta)} \nabla \cdot \mathbf{u}_s - \alpha^{(\theta)} M^{(\theta)} \xi\right),$$
$$p_f(\mathbf{u}) = -\alpha^{(\theta)} M^{(\theta)} \nabla \cdot \mathbf{u}_s + M^{(\theta)} \xi, \qquad \theta = b, f.$$

Here $G^{(\theta)}$ and $\lambda_U^{(\theta)} = K_U^{(\theta)} - \frac{2}{3} G^{(\theta)}$ and $\alpha^{(\theta)}$ are the Lamé coefficients ($K_U^{(\theta)}$ is the closed bulk modulus) and $\alpha^{(\theta)}$ is the effective stress coefficient.

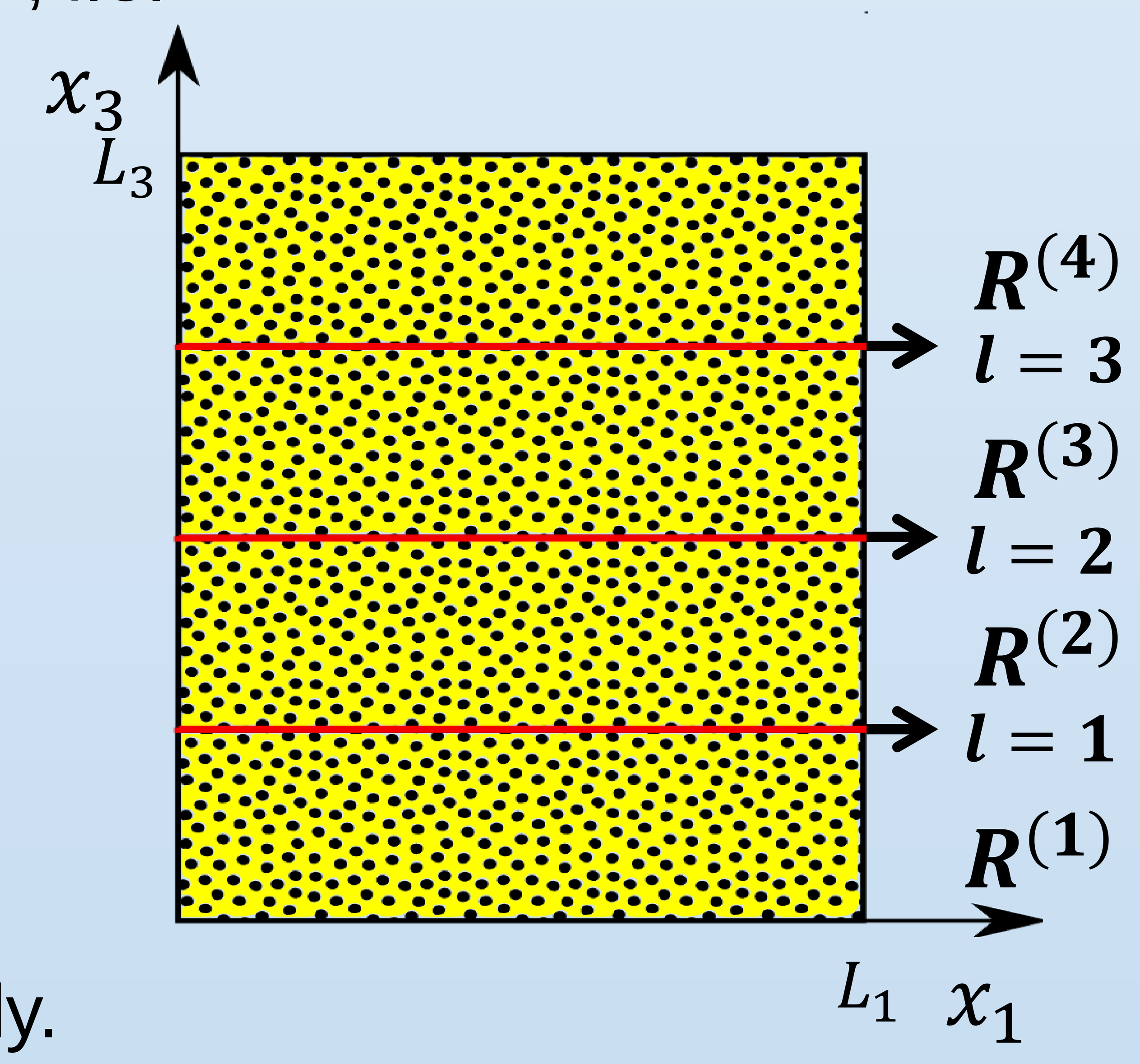
The boundary conditions at a fracture inside a Biot medium

Consider a rectangular domain $\Omega = (0, L_1) \times (0, L_3)$ with boundary Γ in the (x_1, x_3)- plane, with x_1 and x_3 being the horizontal and vertical coordinates, respectively.

Let us assume that the domain Ω contains a set of $J^{(f)}$ horizontal fractures $\Gamma^{(f,l)}$, $l = 1, \cdots, J^{(f)}$ each one of length L_1 and aperture $h^{(f)}$. This set of fractures divides Ω in a collection of non-overlapping rectangles $R^{(l)}$, $l = 1, \cdots, J^f + 1$, so that

$$\Omega = \cup_{l=1}^{J^{(f)}+1} R^{(l)}.$$

Consider a fracture $\Gamma^{(f,l)}$ and the two rectangles $R^{(l)}$ and $R^{(l+1)}$ having as a common side $\Gamma^{(f,l)}$. Let $\nu_{l,l+1}$ and $\chi_{l,l+1}$ be the unit outer normal and a unit tangent (oriented counterclockwise) on $\Gamma^{(f,l)}$ from $R^{(l)}$ to $R^{(l+1)}$, such that $\{\nu_{l,l+1}, \chi_{l,l+1}\}$ is an orthonormal system on $\Gamma^{(f,l)}$. Let $[\mathbf{u}_s]$, $[\mathbf{u}_f]$ denote the jumps of the solid and fluid displacement vectors at $\Gamma^{(f,l)}$, i.e.



$$[\mathbf{u}_s] = \left(\mathbf{u}_s^{(l+1)} - \mathbf{u}_s^{(l)}\right)|_{\Gamma^{(f,l)}},$$

where $\mathbf{u}_s^{(l)} \equiv \mathbf{u}_s|_{R^{(l)}}$ denotes the trace of $\mathbf{u}_s$ as seen from to $R^{(l)}$.

The boundary conditions at $\Gamma^{(f,l)}$, derived in Nakagawa and Schoenberg (2007), are defined in terms of the fracture normal and tangencial compliances η_N and η_T, respectively. Besides imposing stress continuity along $\Gamma^{(f,l)}$, they can be stated as follows:

$$[\mathbf{u}_s \cdot \boldsymbol{\nu}_{l,l+1}] =$$
$$\eta_N \left(\left(1 - \alpha^{(f)} \tilde{B}^{(f)} (1 - \Pi)\right) \boldsymbol{\sigma}(\mathbf{u}) \nu_{l,l+1} \cdot \boldsymbol{\nu}_{l,l+1} - \alpha^{(f)} \frac{1}{2} \left(\left(-p_f^{(l+1)}\right) + \left(-p_f^{(l)}\right)\right) \Pi\right), \qquad \Gamma^{(f,l)},$$

$$[\mathbf{u}_s \cdot \boldsymbol{\chi}_{l,l+1}] = \eta_T \boldsymbol{\sigma}(\mathbf{u}) \nu_{l,l+1} \cdot \boldsymbol{\chi}_{l,l+1}, \qquad \Gamma^{(f,l)},$$

$$[\mathbf{u}_f \cdot \boldsymbol{\nu}_{l,l+1}] = \alpha^{(f)} \eta_N \left(\boldsymbol{\sigma}(\mathbf{u}) \nu_{l,l+1} \cdot \boldsymbol{\nu}_{l,l+1} + \frac{1}{\tilde{B}^{(f)}} \frac{1}{2} \left(\left(-p_f^{(l+1)}\right) + \left(-p_f^{(l)}\right)\right)\right) \Pi, \qquad \Gamma^{(f,l)},$$

$$\left(-p_f^{(l+1)}\right) - \left(-p_f^{(l)}\right) = \frac{\mathrm{i}\omega\mu^{(f)}\Pi}{\hat{\kappa}^{(f)}} \frac{1}{2} \left(\mathbf{u}_f^{(l+1)} + \mathbf{u}_f^{(l)}\right) \cdot \boldsymbol{\nu}_{l,l+1}, \qquad \Gamma^{(f,l)},$$

where $\hat{\kappa}^{(f)} = \frac{\kappa^{(f)}}{h^{(f)}}$, $h^{(f)}$ is the fracture aperture and $\mu^{(f)}$ is the fluid viscosity in the fractures. The fracture dry plane wave modulus $H_m^{(f)} = K_m^{(f)} + \frac{4}{3} G^{(f)}$ and the dry fracture shear modulus $G^{(f)}$ are defined by the relations $\eta_N = \frac{h^{(f)}}{H_m^{(f)}}$, $\eta_T = \frac{h^{(f)}}{G^{(f)}}$.

Furthermore, $\epsilon = \frac{(1+\mathrm{i})}{2} \left(\frac{\omega\mu^{(f)}\alpha^{(f)}\eta_N}{2\tilde{B}^{(f)}\hat{\kappa}^{(f)}}\right)^{1/2}$, $\Pi(\epsilon) = \frac{\tanh(\epsilon)}{\epsilon}$, $\tilde{B}^{(f)} = \frac{\alpha^{(f)} M^{(f)}}{H_U^{(f)}}$.

Denoting by $\omega = 2\pi f$ the angular frequency, Biot's equations of motion in the diffusive range, stated in the space-frequency domain and in the absence of external sources are:

$$\nabla \cdot \boldsymbol{\sigma}(\mathbf{u}) = 0, \quad \frac{\mathrm{i}\omega\mu}{\kappa} \mathbf{u}^f + \nabla p_f(\mathbf{u}) = 0,$$

where μ is the fluid viscosity, κ is the frame permeability and $\mathrm{i} = \sqrt{-1}$.

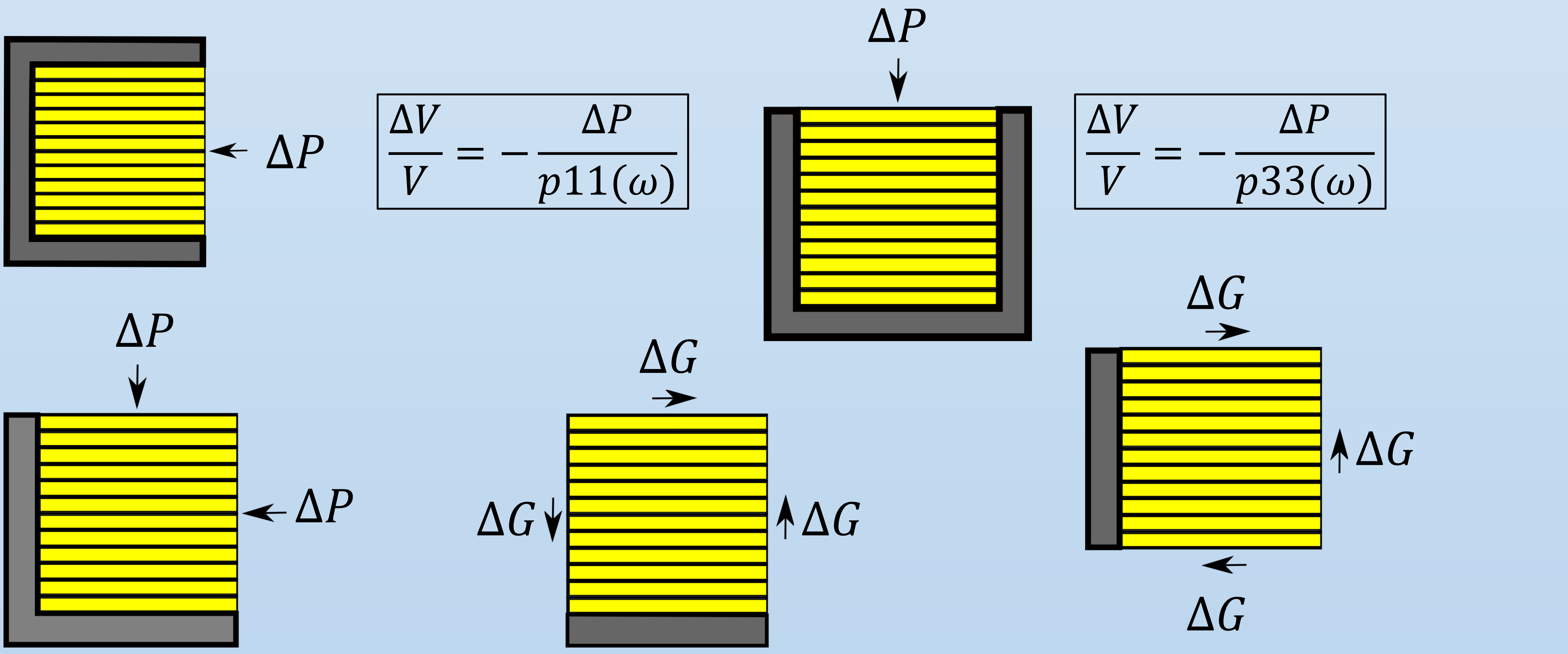
The equivalent TIV medium

Let us consider x_1 and x_3 as the horizontal and vertical coordinates, respectively. Following Gelinsky and Shapiro (1997), and Krzikalla and Müller (2011) an horizontally fractured Biot medium behaves as a TIV medium with a vertical symmetry axis at long wavelengths.

Denoting by $\tau_{ij}(\tilde{\mathbf{u}}_s)$ and $\epsilon_{ij}(\tilde{\mathbf{u}}_s)$ the stress and strain tensor components of the equivalent TIV medium, where $\tilde{\mathbf{u}}_s$ denotes the solid displacement vector at the macroscale, the stress-strain relations, stated in the space-frequency domain, are (Carcione, 1992; 2015):

$$
\begin{aligned}
\tau_{11}(\tilde{\mathbf{u}}_s) &= p_{11}\epsilon_{11}(\tilde{\mathbf{u}}_s) + p_{12}\epsilon_{22}(\tilde{\mathbf{u}}_s) + p_{13}\epsilon_{33}(\tilde{\mathbf{u}}_s),\\
\tau_{22}(\tilde{\mathbf{u}}_s) &= p_{12}\epsilon_{11}(\tilde{\mathbf{u}}_s) + p_{11}\epsilon_{22}(\tilde{\mathbf{u}}_s) + p_{13}\epsilon_{33}(\tilde{\mathbf{u}}_s),\\
\tau_{33}(\tilde{\mathbf{u}}_s) &= p_{13}\epsilon_{11}(\tilde{\mathbf{u}}_s) + p_{13}\epsilon_{22}(\tilde{\mathbf{u}}_s) + p_{33}\epsilon_{33}(\tilde{\mathbf{u}}_s),\\
\tau_{23}(\tilde{\mathbf{u}}_s) &= 2p_{55}\epsilon_{23}(\tilde{\mathbf{u}}_s),\\
\tau_{13}(\tilde{\mathbf{u}}_s) &= 2p_{55}\epsilon_{13}(\tilde{\mathbf{u}}_s),\\
\tau_{12}(\tilde{\mathbf{u}}_s) &= 2p_{66}\epsilon_{12}(\tilde{\mathbf{u}}_s).
\end{aligned}
$$

The p_{IJ} are the complex and frequency-dependent Voigt stiffnesses were determined with a generalization of the time-harmonic experiments presented in Picotti et al. (2010) and Santos et al. (2012). Each harmonic experiment is associated with a boundary value problem for the diffusive Biot's equations of motion, which solution is obtained using the FE method.



$$\frac{\Delta V}{V} = -\frac{\Delta P}{p11(\omega)}$$

$$\frac{\Delta V}{V} = -\frac{\Delta P}{p33(\omega)}$$

$$p_{13}(\omega) = \frac{p_{11}\epsilon_{11} - p_{33}\epsilon_{33}}{\epsilon_{11} - \epsilon_{33}}$$

$$\tan[\theta(\omega)] = -\frac{\Delta G}{p55(\omega)}$$

$$\tan[\theta(\omega)] = -\frac{\Delta G}{p66(\omega)}$$

In order to relate the pore and confining pressure to the fracture compliances η_N, η_T, following Brajanovski et al. (2005), Daley et al. (2006) and Carcione et al. (2012), let us define the new compliances (with units of 1/Pa)

$$Z_N = \frac{\eta_N}{L}, \qquad Z_T = \frac{\eta_T}{L}$$

characterizing the fractures, where L is the fracture distance. The compliances Z_N and Z_T are assumed to be dependent on the effective stress $\sigma = p_c - p$, where p_c is the confining pressure and p the pore pressure as

$$Z_q = Z_{q\infty} + \left(Z_{q0} - Z_{q\infty}\right)e^{-\sigma/\tau_q} \qquad q = N, T,$$

where Z_{q0}, $Z_{q\infty}$ and τ_q are constants.

Numerical experiments

The FE procedure was used to determine the complex stiffnesses $p_{IJ}(\omega)$; the associated energy velocities and dissipation coefficients were computed as in Carcione (2015).

To analyze the pore pressure effect on velocities and attenuation, we choose a square sample of side length 10 m, with a fracture distance $L = 1$ m. The background is homogeneous and isotropic, with grains properties $K_s = 37$ GPa, $\mu_s = 44$ GPa and $\rho_s = 2650$ kg/m^3 (Carcione et al., 2013). The dry-rock bulk and shear moduli are given by the Krief model (Krief et al., 1990):

$$\frac{K_m}{K_s} = \frac{\mu}{\mu_s} = (1-\phi)^{3/(1-\phi)}.$$

Porosity ϕ and permeability κ are 0.25 and 0.25 D in the background and 0.75 and 60.8 D in the fractures, respectively. Porosity and permeability are related by the equation (Carcione et al., 2000):

$\kappa = \frac{r_g^2 \phi^3}{45(1-\phi)^2}$, where $r_g = 20$ μm denotes the average radius of the grains.

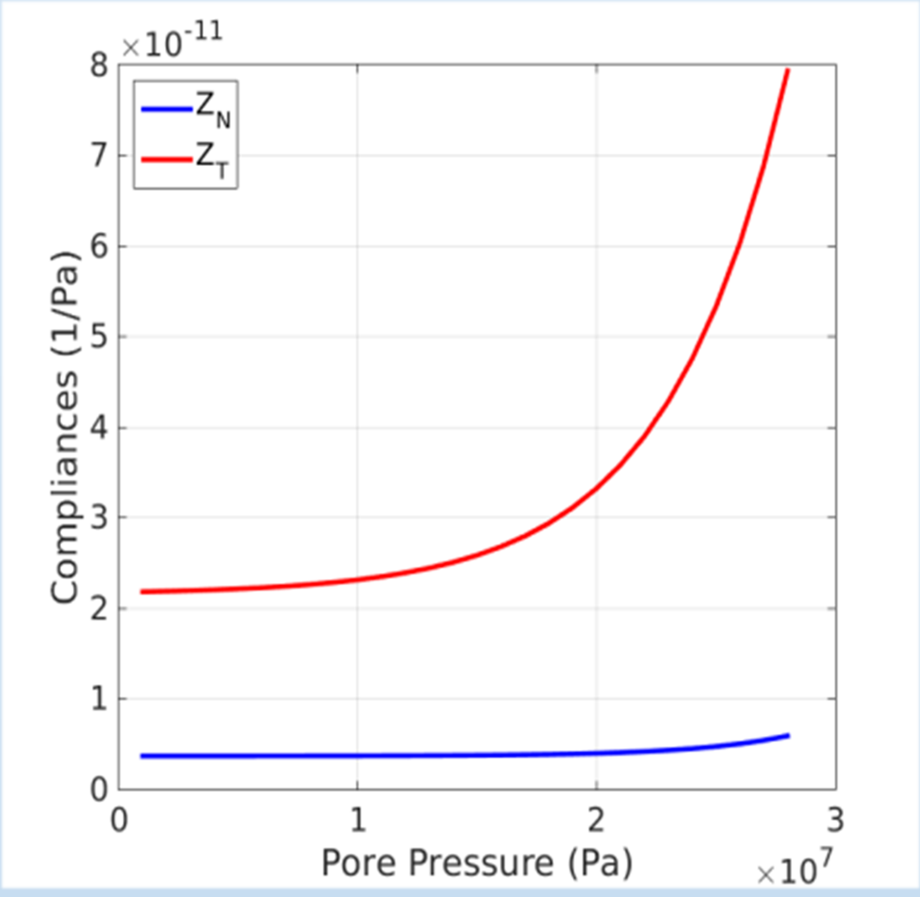
The saturant fluid is brine with density 1040 kg/m^3, bulk modulus 2.25 GPa and viscosity 0.003 Pa·s. The compliances vary as in Daley et al. (2006) as follows:

$$G_b Z_{N_0} = 1.5, \quad (\lambda_{U,b} + 2G_b) Z_{T_0} = 0.25.$$

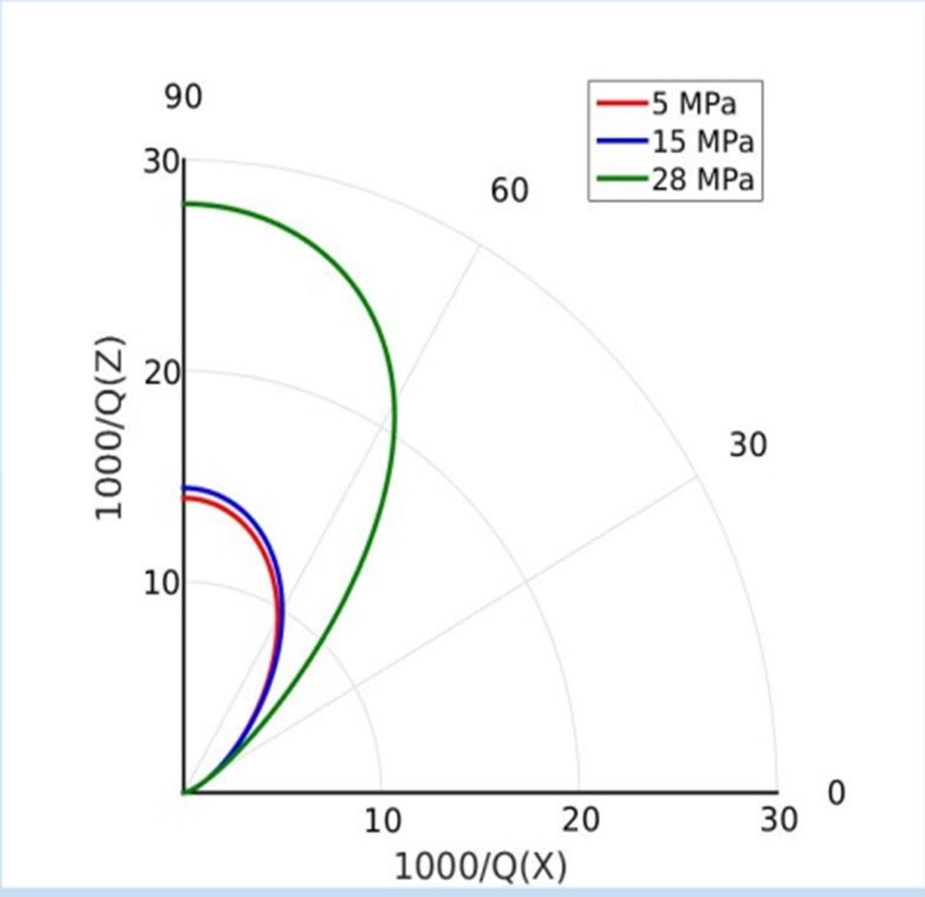
Also,

$$Z_{T\infty} = \frac{Z_{T_0}}{5},\ Z_{N\infty} = \frac{Z_{N_0}}{2},\ \tau_T = 5\ \text{MPa},\ \tau_N = 4\ \text{MPa}.$$

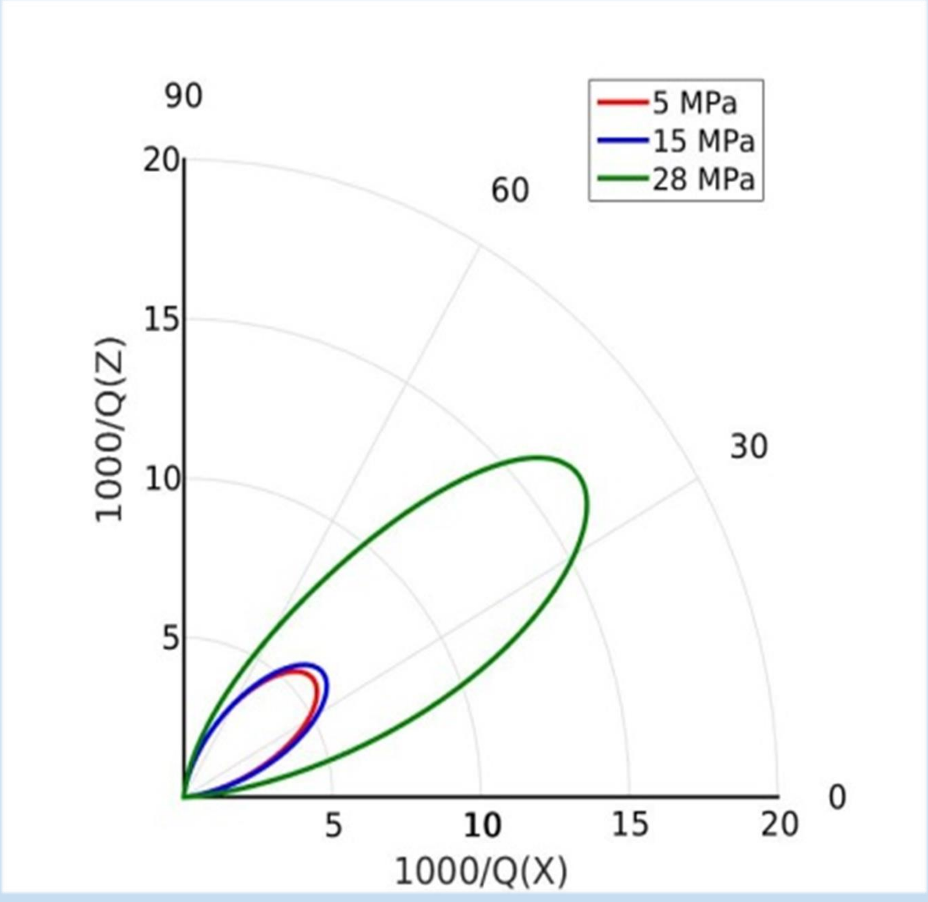
Let us consider a constant confining pressure p_c =30 MPa and three pore pressures 5, 15 and 28 MPa, normal, middle and overpressure values, respectively.



Compliances as a function of pore pressure.

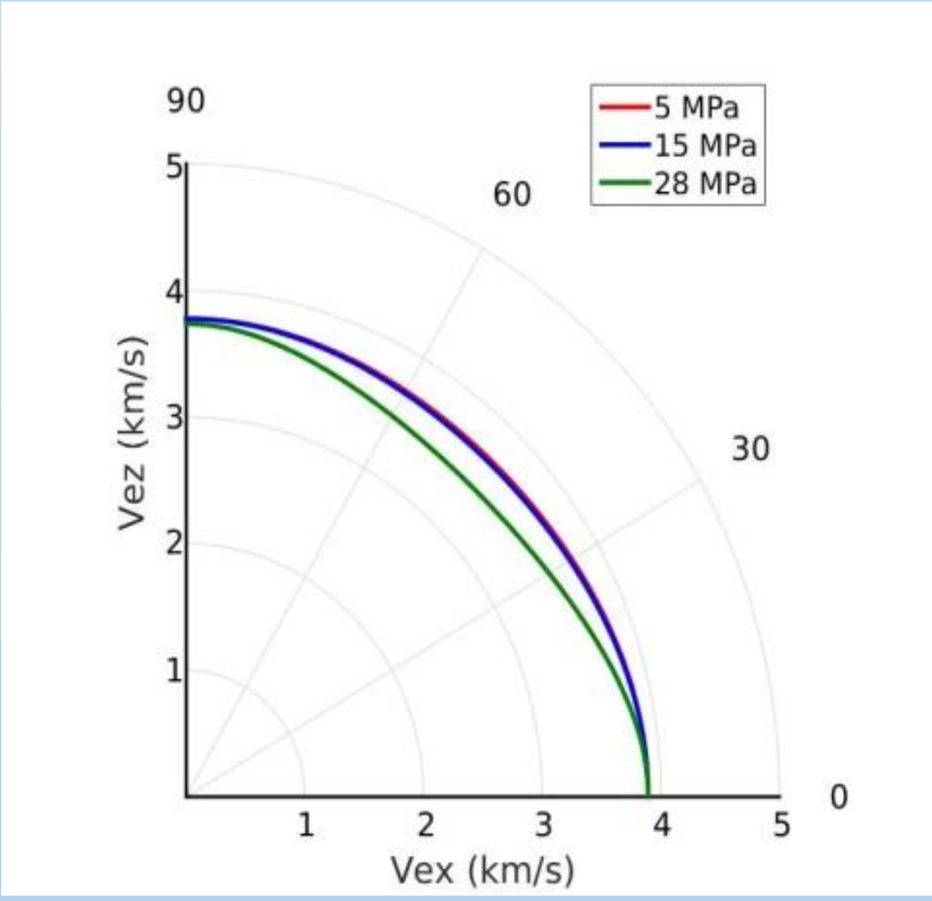


(a) qP-Waves

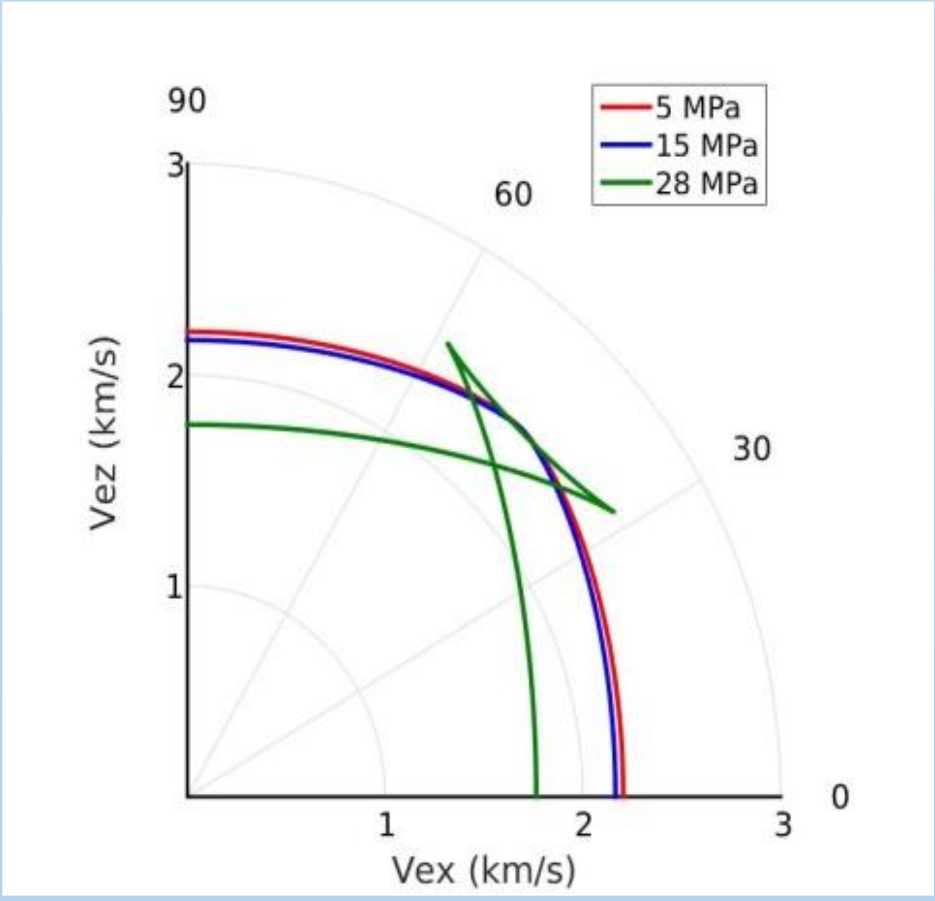


(b) qSV-Waves

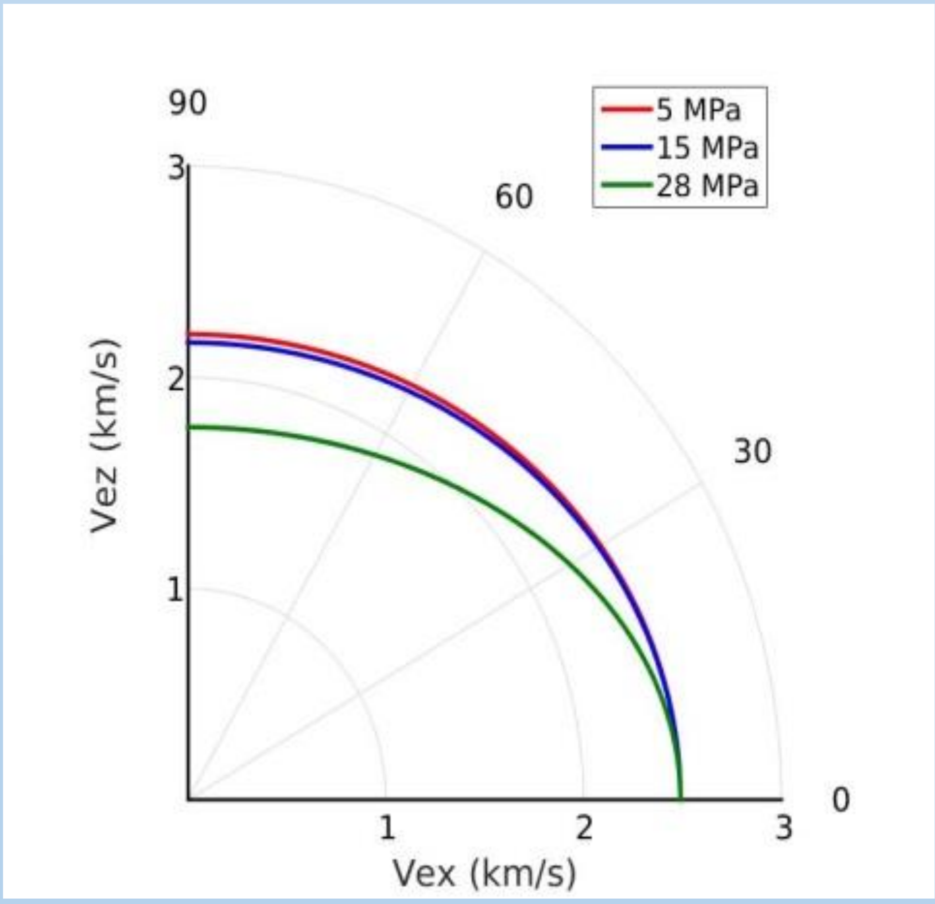
Dissipation factor at 60Hz for fractures with normal pore pressure (red line, 5 MPa), middle pressure (blue line, 15 MPa) and overpressure (green line, 28 MPa). Fractures are represented using the boundary conditions.



(a) qP-Waves



(b) qSV-Waves



(c) SH-Waves

Polar representation of the energy velocity vector at 60 Hz for fractures with normal pore pressure (red line, 5 MPa), middle pressure (blue line, 15 MPa) and overpressure (green line, 28 MPa). Fractures are represented using the boundary conditions.

Conclusions

This work used a finite element procedure to determine the five complex and frequency-dependent stiffnesses of the TIV medium equivalent to a horizontally highly heterogeneous fractured Biot medium, with fractures represented as boundary conditions. The procedure was applied to analyze the sensitivity of velocities and attenuation to variations in the pore pressure. The numerical experiments show that attenuation and velocity anisotropy is enhanced as pore pressure increases. Consequently, pore pressure is an important variable in the study and characterization of fractured reservoirs.