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Relative Orbital Elements: Theory and Applications

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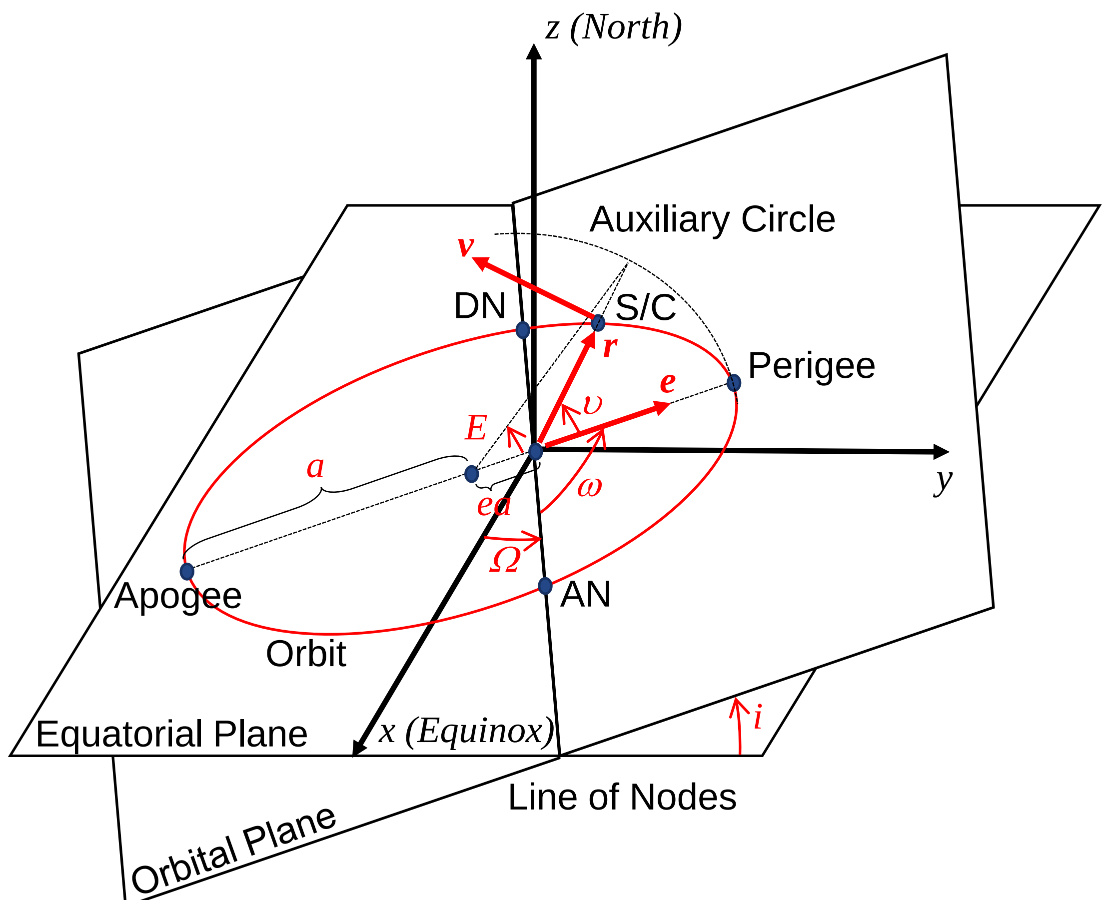
Parameterization of Absolute Orbit Motion

- Inertial Position and Velocity

$$\boldsymbol{x} = \begin{pmatrix} r_x \\ r_y \\ r_z \\ v_x \\ v_y \\ v_z \end{pmatrix}$$

- Absolute Orbital Elements

$$\boldsymbol{\alpha} = \begin{pmatrix} a \\ u \\ e_x \\ e_y \\ i \\ \Omega \end{pmatrix} = \begin{pmatrix} a \\ \omega + M \\ e\ \cos\omega \\ e\ \sin\omega \\ i \\ \Omega \end{pmatrix}$$



Note: Relationship between $\boldsymbol{x}$ and $\boldsymbol{\alpha}$ is straightforward from 2-body problem

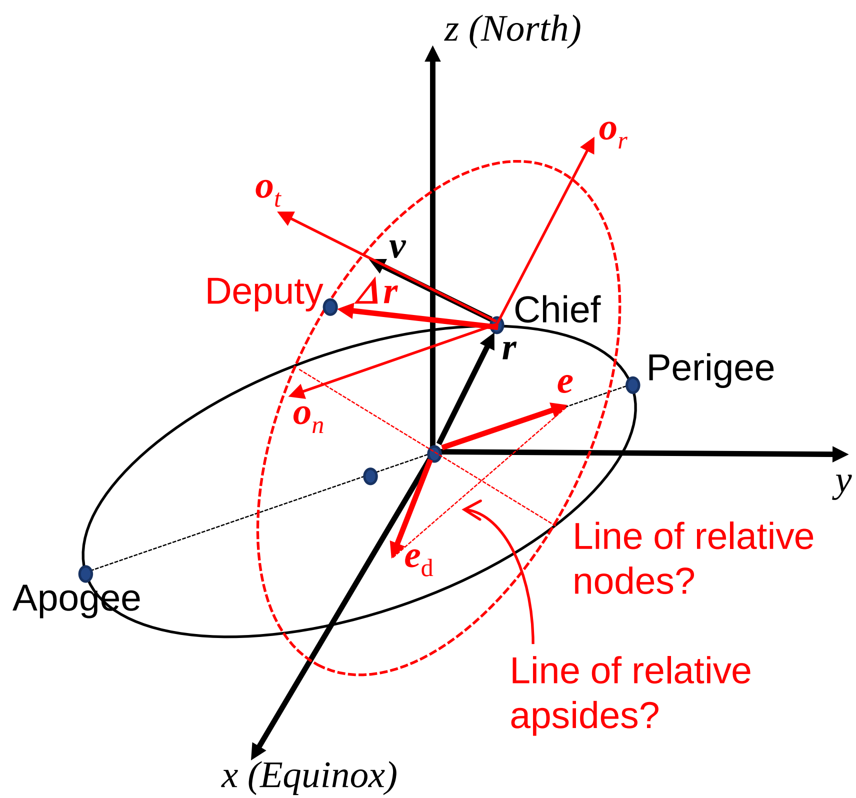
Parameterization of Relative Orbit Motion

- Hill Coordinates (RTN)

$$\delta \boldsymbol{x} = \begin{pmatrix} \delta r_r \\ \delta r_t \\ \delta r_n \\ \delta v_r \\ \delta v_t \\ \delta v_n \end{pmatrix} = \begin{pmatrix} \Delta \boldsymbol{r} \cdot \boldsymbol{o_r} \\ \Delta \boldsymbol{r} \cdot \boldsymbol{o_t} \\ \Delta \boldsymbol{r} \cdot \boldsymbol{o_n} \\ \Delta \boldsymbol{v} \cdot \boldsymbol{o_r} + \Delta \boldsymbol{r} \cdot \dot{\boldsymbol{o}}_{\boldsymbol{r}} \\ \Delta \boldsymbol{v} \cdot \boldsymbol{o_t} + \Delta \boldsymbol{r} \cdot \dot{\boldsymbol{o}}_{\boldsymbol{t}} \\ \Delta \boldsymbol{v} \cdot \boldsymbol{o_n} + \Delta \boldsymbol{r} \cdot \dot{\boldsymbol{o}}_{\boldsymbol{n}} \end{pmatrix}$$

- Relative Orbital Elements (ROE)

$$\delta \boldsymbol{\alpha} = \begin{pmatrix} \delta a \\ \delta \lambda \\ \delta e_x \\ \delta e_y \\ \delta i_x \\ \delta i_y \end{pmatrix} = \begin{pmatrix} (a_d - a)/a \\ (u_d - u) + (\Omega_d - \Omega)\cos i \\ e_{x_d} - e_x \\ e_{y_d} - e_y \\ i_d - i \\ (\Omega_d - \Omega)\sin i \end{pmatrix}$$



Questions:

- Can we directly map $\delta \boldsymbol{x}$ and $\delta \boldsymbol{\alpha}$?
- What is the geometry of the relative motion?

Linear Coordinate Mapping (1)

- Assumption of small Relative Orbital Elements (ROE)

Note: Not the individual components, but their sum!

$$\delta r / r \ll 1 \quad \Leftrightarrow \quad \delta\alpha_i \ll 1 \quad \Rightarrow \quad \delta u \equiv \Delta u = \Delta M + \Delta\omega \ll 1$$

- The inertial position of the deputy $\boldsymbol{r}_\mathrm{d}$ can be expressed in the RTN frame *C* centered on the chief spacecraft

$$^{C}\boldsymbol{r}_d = {}^{C}(r + \delta r_r, \delta r_t, \delta r_n)^T = \boldsymbol{T}^{CD}\,{}^{D}(r_d, 0, 0)^T$$

Substitute back neglecting 2nd order terms of the form δ^2

- The rotation matrix $\boldsymbol{T}^{CD}$ (from deputy's to chief's RTN frame) and the deputy position magnitude r_d can be expanded to first order

$$\boldsymbol{T}^{CD} = \boldsymbol{T}^{CI}\boldsymbol{T}^{ID} \approx \boldsymbol{T}^{CI}(\boldsymbol{T}^{IC} + \delta\boldsymbol{T}^{IC}) = \boldsymbol{I}_{3x3} + \boldsymbol{T}^{CI}\delta\boldsymbol{T}^{IC}$$

$$r_d \approx r + \delta r$$

Linear Coordinate Mapping (2)

- The direction cosine matrix $\boldsymbol{T}^{\mathrm{CI}}$ from the inertial to the chief's RTN frame is a function of the chief's orbital elements only

True argument of latitude

$$\boldsymbol{T}^{CI} = \boldsymbol{T}^{CI}(\Omega, i, f = \upsilon + \omega)$$

True anomaly

- The direct mapping between RTN frame position coordinates and orbital element differences yields to first-order

$$\begin{aligned}
\delta r_r &\approx (r/a)\Delta a + (ae\sin\nu/\eta)\Delta M - a\cos\nu\Delta e \\
\delta r_t &\approx (r/\eta^3)(1+e\cos\nu)^2\Delta M + r\Delta\omega + (r\sin\nu/\eta^2)(2+e\cos\nu)\Delta e + r\Delta\Omega\cos i \\
\delta r_n &\approx r(\sin(\nu+\omega)\Delta i - \cos(\nu+\omega)\Delta\Omega\sin i)
\end{aligned}$$

Eccentricity factor

$$\eta = \sqrt{1-e^2}$$

- These equations are valid for arbitrary eccentricity and similar expressions can be derived for the relative velocity after differentiation

Linear Coordinate Mapping (3)

- The linear mapping can be reduced to a more convenient form under the additional assumption of near-circular chief orbit

$$\delta\alpha \ll 1 \quad \text{and} \quad e \ll 1$$

- In particular we can now drop 2nd order terms of the form δ^2, e^2, and $\delta \cdot e$, and rearrange the equations to show explicitly our ROE

$$\begin{array}{lclllll} \delta r_r/a & \approx & \delta a & & -\delta e_x \cos u & -\delta e_y \sin u \\ \delta r_t/a & \approx & -\frac{3}{2}\delta a u & +\delta\lambda & +2\delta e_x \sin u & -2\delta e_y \cos u \\ \delta r_n/a & \approx & & & +\delta i_x \sin u & -\delta i_y \cos u \end{array}$$

From mean motion (2-body problem)

$$\Delta \dot{u} = \frac{d(\Delta u)}{dt} = -\frac{3}{2}\sqrt{\frac{\mu}{a^5}}\Delta a = -\frac{3}{2}n\frac{\Delta a}{a}$$

$$nt_0 = u_0 = 0$$

This is nothing but the solution of the HCW equations

Relative Orbital Elements as Hill's Integration Constants

- We have found a 1:1 correspondence between the solution of the HCW equations and the linear coordinate mapping. This is no surprise since we are working under *similar* assumptions.

- The integration constants of the HCW equations are the relative orbital elements (ROE), which can be expressed through a polar representation

Relative Perigee

$$\delta\boldsymbol{e} = \begin{pmatrix} \delta e_x \\ \delta e_y \end{pmatrix} = \delta e \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix}$$

$$\delta\boldsymbol{i} = \begin{pmatrix} \delta i_x \\ \delta i_y \end{pmatrix} = \delta i \begin{pmatrix} \cos\vartheta \\ \sin\vartheta \end{pmatrix}$$

Relative Ascending Node

$$\Rightarrow \quad \begin{aligned} \delta r_r/a &= \delta a & & -\delta e\cos(u-\varphi) \\ \delta r_t/a &= \delta\lambda & -\tfrac{3}{2}\delta a u & +2\delta e\sin(u-\varphi) \\ \delta r_n/a &= & & +\delta i\sin(u-\vartheta) \end{aligned}$$

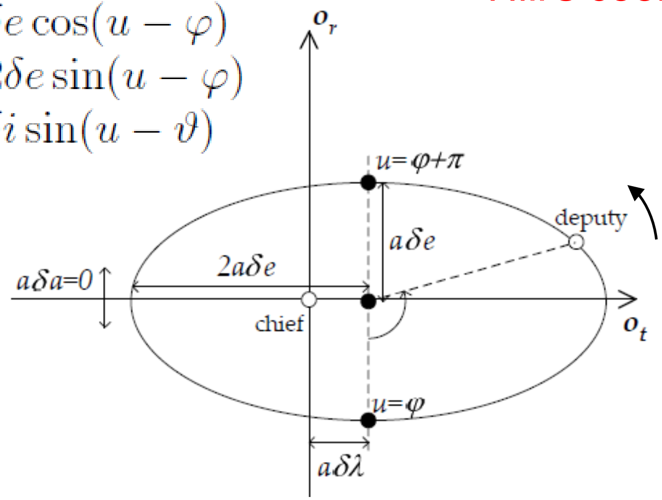
Radial and along-track offsets

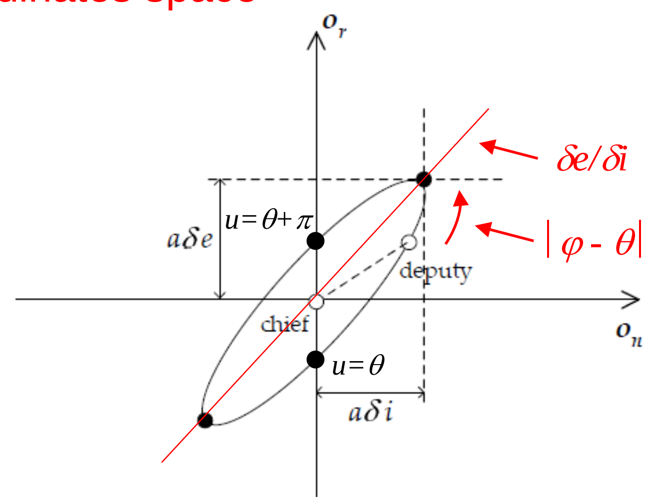
Periodic in- and out-of-plane oscillations

Geometry of Relative Motion through ROE

$$\begin{aligned} \delta r_r/a &= \delta a & & -\delta e\cos(u-\varphi) \\ \delta r_t/a &= \delta\lambda & -\tfrac{3}{2}\delta a u & +2\delta e\sin(u-\varphi) \\ \delta r_n/a &= & & +\delta i\sin(u-\vartheta) \end{aligned}$$

Hill's coordinates space

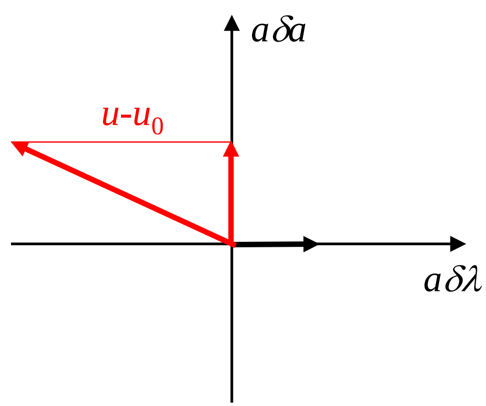


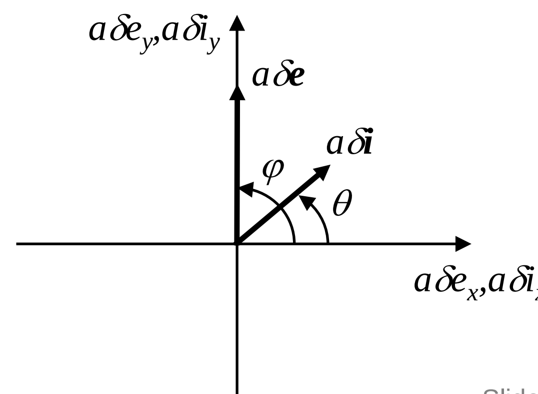


$$\delta\boldsymbol{e} = \begin{pmatrix} \delta e_x \\ \delta e_y \end{pmatrix} = \delta e \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix}$$

$$\delta\boldsymbol{i} = \begin{pmatrix} \delta i_x \\ \delta i_y \end{pmatrix} = \delta i \begin{pmatrix} \cos\vartheta \\ \sin\vartheta \end{pmatrix}$$

Relative orbital elements space





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Relative Eccentricity/Inclination Vector Separation (1)

- Fact: The mean along-track relative position δr_t or longitude $\delta\lambda$ is the most difficult to estimate, predict, and control due to uncertainties (e.g., atmosphere, propulsion) in combination with Kepler's equation

- Idea: Adopt the inter-satellite distance in the plane perpendicular to the flight direction (RN) as a measure of the collision risk, regardless of the along-track separation

- Question: Can we guarantee a minimum separation perpendicular to the flight direction at all times? Can this be done through a proper selection of the relative eccentricity and inclination vectors?

$$\begin{pmatrix}\delta r_n \\ \delta r_r\end{pmatrix} = \boldsymbol{M}\begin{pmatrix}\cos u \\ \sin u\end{pmatrix} \quad \text{with} \quad \boldsymbol{M} = \begin{bmatrix}-\delta i_y & +\delta i_x \\ -\delta e_x & -\delta e_y\end{bmatrix}$$

Tilted ellipse in the RN plane (for $\delta a = 0$)

Relative Eccentricity/Inclination Vector Separation (2)

- The semi-minor and semi-major axes of a tilted ellipse are given by the square roots of the two eigenvalues of $\boldsymbol{M}^{\mathrm{T}}\boldsymbol{M}$ where

$$\lambda_1\lambda_2 = \det\left(\boldsymbol{M}^{T}\boldsymbol{M}\right) = a^4(\delta\boldsymbol{e}\cdot\delta\boldsymbol{i})^2 \qquad \lambda_1+\lambda_2 = \mathrm{trace}\left(\boldsymbol{M}^{T}\boldsymbol{M}\right) = a^2(\delta e^2+\delta i^2)^2$$

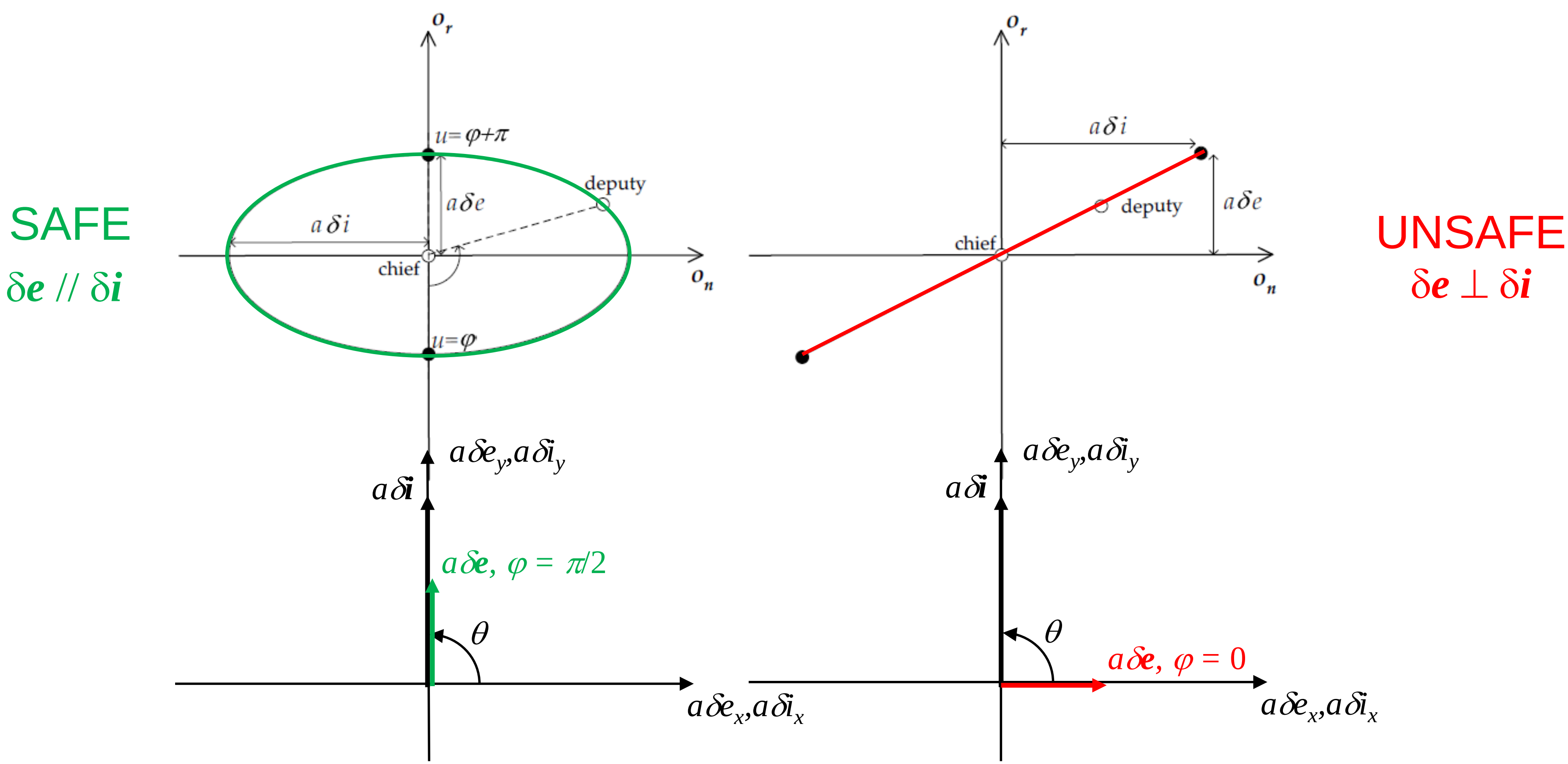
- A lower threshold for the separation perpendicular to the flight direction is given for bounded relative motion by

$$\delta r_{nr}^{\min} = \frac{\sqrt{2}a\left|\delta\boldsymbol{e}\cdot\delta\boldsymbol{i}\right|}{\left(\delta e^2+\delta i^2+\left|\delta\boldsymbol{e}+\delta\boldsymbol{i}\right|\cdot\left|\delta\boldsymbol{e}-\delta\boldsymbol{i}\right|\right)^{1/2}}$$

- Given the amplitudes of $\delta\boldsymbol{e}$ and $\delta\boldsymbol{i}$, the maximum separation or minimum collision risk is given by parallel or anti-parallel relative eccentricity and inclination vectors

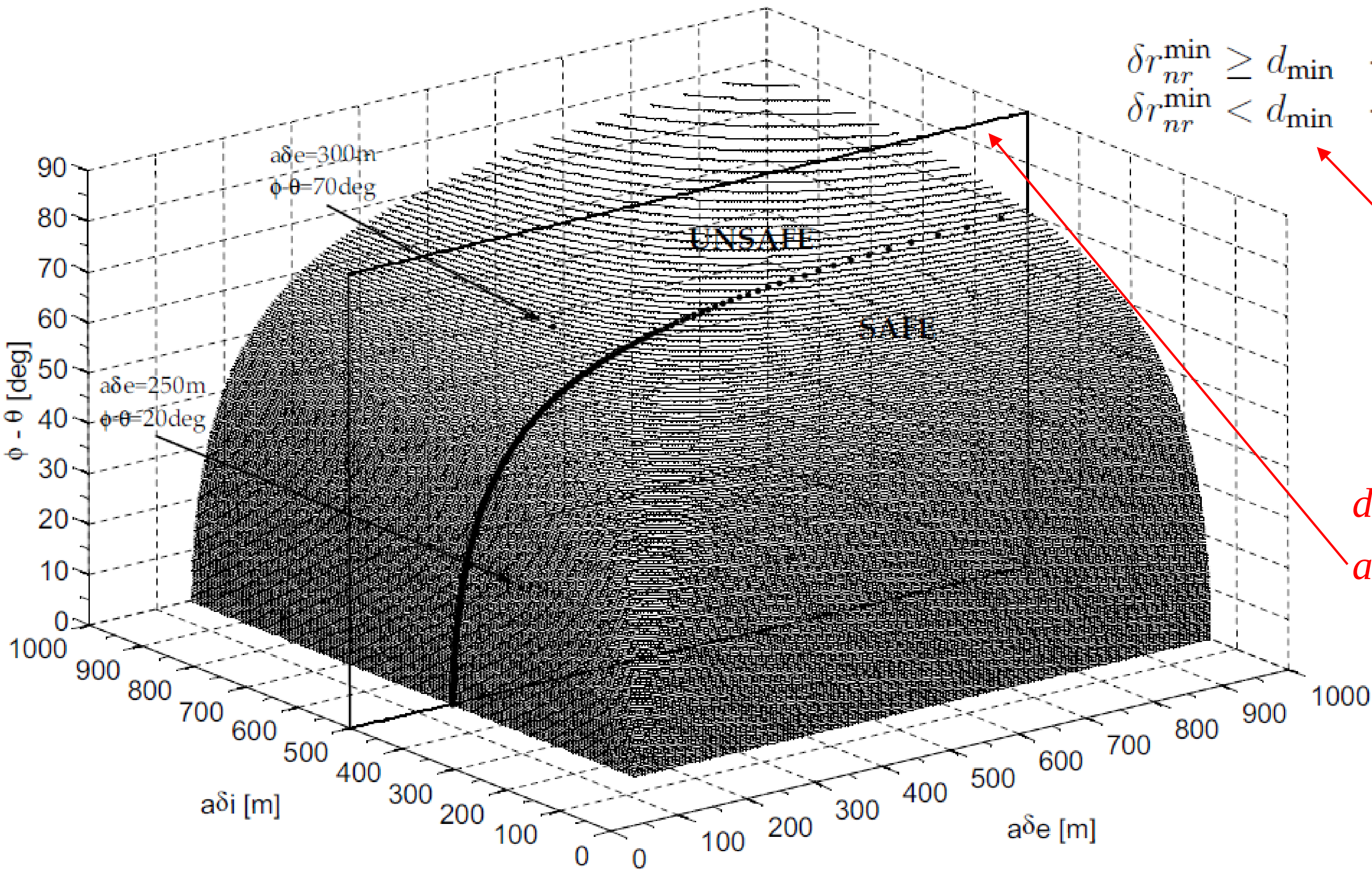
Relative Eccentricity/Inclination Vector Separation (3)

$$\delta r_{nr}^{\min}(u = \varphi, \varphi + k\pi) = a \min\{\delta e, \delta i\}$$



Collision-Free Formation-Flying Configurations

$$\delta r_{nr}^{\min} \geq d_{\min} \Leftrightarrow \text{SAFE}$$
$$\delta r_{nr}^{\min} < d_{\min} \Leftrightarrow \text{UNSAFE}$$



Design parameter

Example

$d_{\min}$ = 150 m

$a\delta i$ = 500 m

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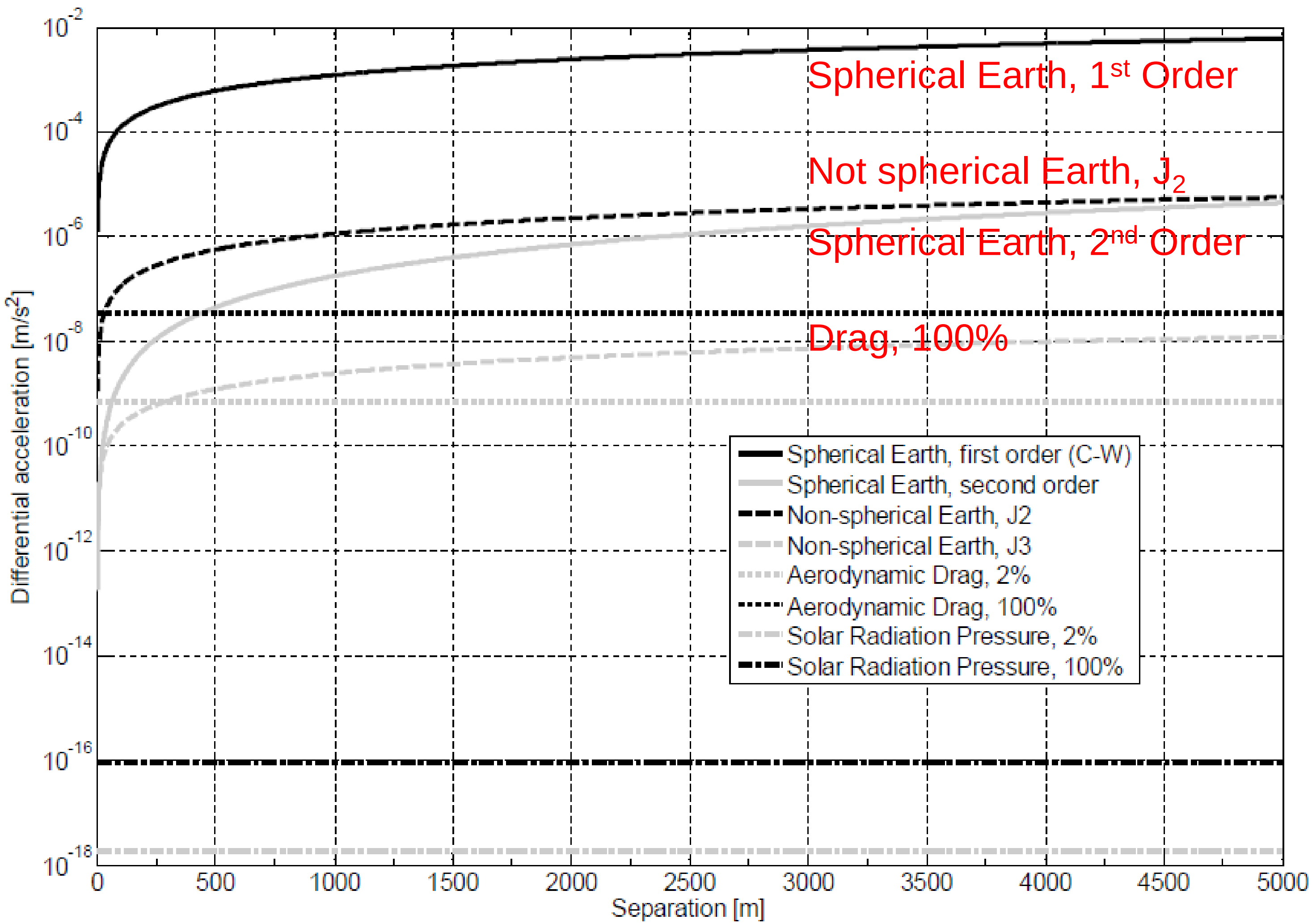
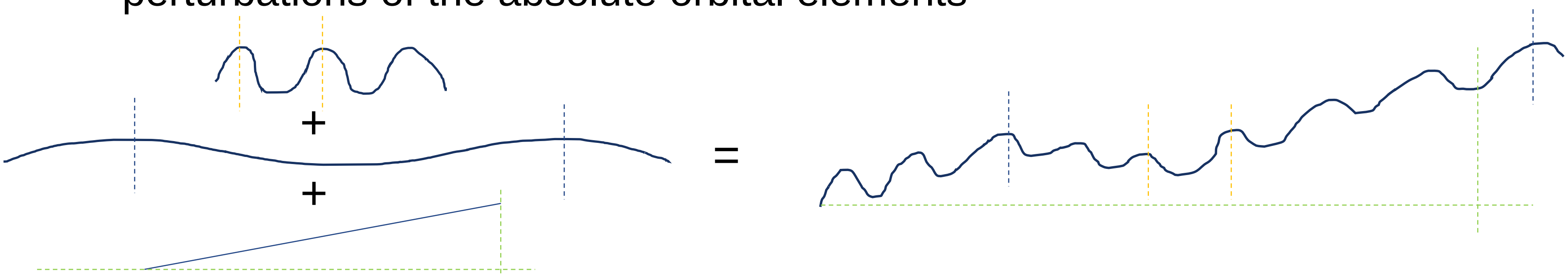
Perturbed Relative Motion (Low Earth Orbit, <1500km)



Perturbed Relative Motion: Earth Oblateness (1)

- The Earth's equatorial bulge causes short-, long-period and secular perturbations of the absolute orbital elements



- The artificial satellite theory of Brouwer and Lyddane [1959-1963] provides the analytical tool to capture these effects for absolute orbital elements

$$\frac{d}{dt}\begin{pmatrix} a \\ e \\ i \\ \Omega \\ \omega \\ M \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ -3\gamma n \cos i \\ \frac{3}{2}\gamma n(5\cos^2 i - 1) \\ \frac{3}{2}\gamma \eta n(3\cos^2 i - 1) \end{pmatrix}$$

Secular variations of Keplerian orbital elements caused by J_2

$$\gamma = \frac{J_2}{2}\left(\frac{R_E}{a}\right)^2\frac{1}{\eta^4}$$

Space rendezvous
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Perturbed Relative Motion: Earth Oblateness (2)

- We can substitute the long-period and secular effects into our definition of ROE and neglect 2nd order effects as done previously to obtain

$$\delta\dot{\boldsymbol{\alpha}} = \begin{pmatrix} 0 \\ -\frac{21}{2}\gamma n \sin(2i)\delta i_x \\ -\frac{3}{2}\gamma n(5\cos^2 i - 1)\delta e_y \\ \frac{3}{2}\gamma n(5\cos^2 i - 1)\delta e_x \\ 0 \\ 3\gamma n \sin^2 i \delta i_x \end{pmatrix}$$

- Using the mean argument of latitude u as independent variable, after integration over u-u_0, we obtain

$$\delta\boldsymbol{\alpha}(t) = \begin{pmatrix} \delta a \\ \delta\lambda - \frac{21}{2}(\gamma \sin(2i)\delta i_x + \frac{1}{7}\delta a)(u(t) - u_0) \\ \delta e \cos(\varphi + \varphi'(u(t) - u_0)) \\ \delta e \sin(\varphi + \varphi'(u(t) - u_0)) \\ \delta i_x \\ \delta i_y + 3\gamma \sin^2 i \delta i_x(u(t) - u_0) \end{pmatrix}$$

Secular variations of relative orbital elements caused by J_2

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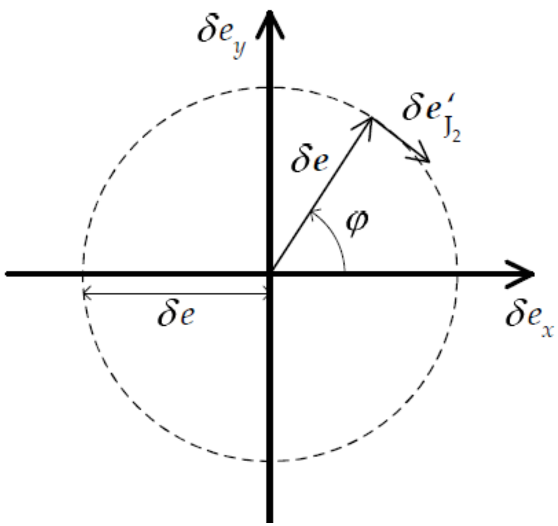
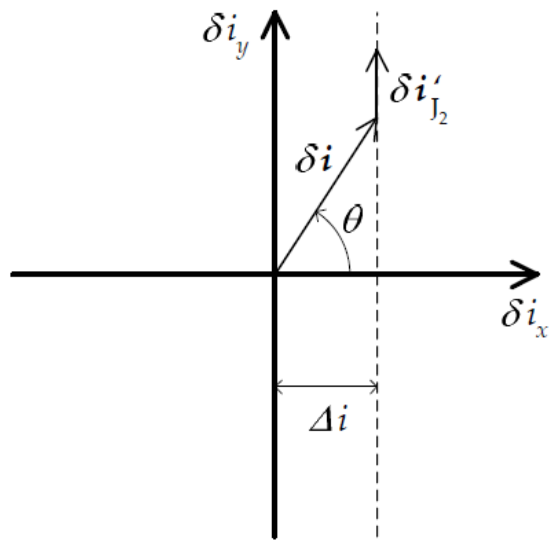
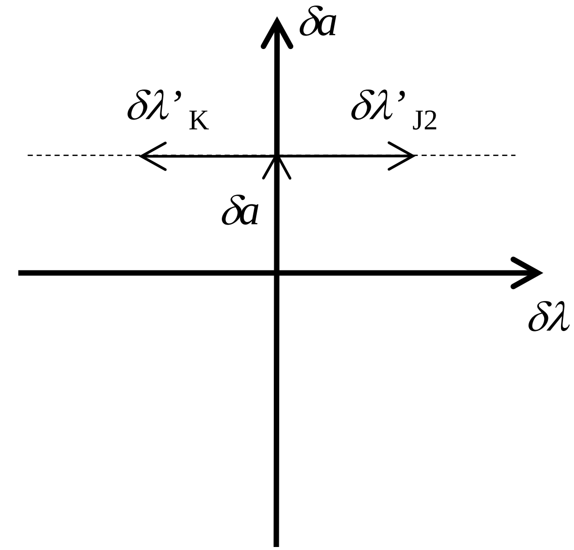
Perturbed Relative Motion: Earth Oblateness (3)



$$\varphi' = \frac{d\varphi}{du} = \frac{3}{2}\gamma(5\cos^2 i - 1)$$



$$\delta i'_{J2} = 3\gamma \sin^2 i\, \delta i_x$$



$$\delta\lambda' = -\frac{21}{2}\left(\gamma \sin(2i)\delta i_x + \frac{1}{7}\delta a\right)$$

Clockwise for sun-synchronous orbits with period of about 100-200 days or about 1000 times the orbital period. A critical inclination exists!

Proportional to Δi and J_2. Note that $\sin(2i)$ is negative for sun-synchronous orbits and closed relative orbits are given by $\Delta a \approx 1000\Delta i$

Perturbed Relative Motion: Differential Drag (1)

- The interaction of the upper atmosphere with the satellite's surface produces the dominant non-conservative disturbance for LEO spacecraft

$$|\ddot{r}_t| = \frac{1}{2}\rho v^2 C_{\mathrm{D}}\frac{A}{m}$$ Along-track acceleration

$$B = C_{\mathrm{D}}\frac{A}{m}$$ Ballistic coefficient

- If we neglect density variations over distances of less than a few kilometers, the relative along-track acceleration for two formation-flying spacecraft is driven by the differential ballistic coefficient ΔB

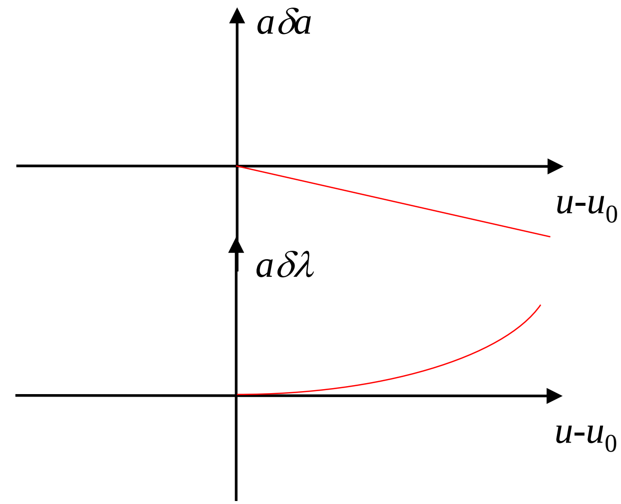
$$\delta r_t = \frac{1}{2}a\Delta\ddot{u}(t-t_0)^2 = \frac{3}{4n^2}\Delta B\rho v^2(u(t)-u_0)^2$$

- The first-order relative motion model can be extended to include this accumulated along-track offset, either using Cartesian or ROE parameters

Perturbed Relative Motion: Differential Drag (2)

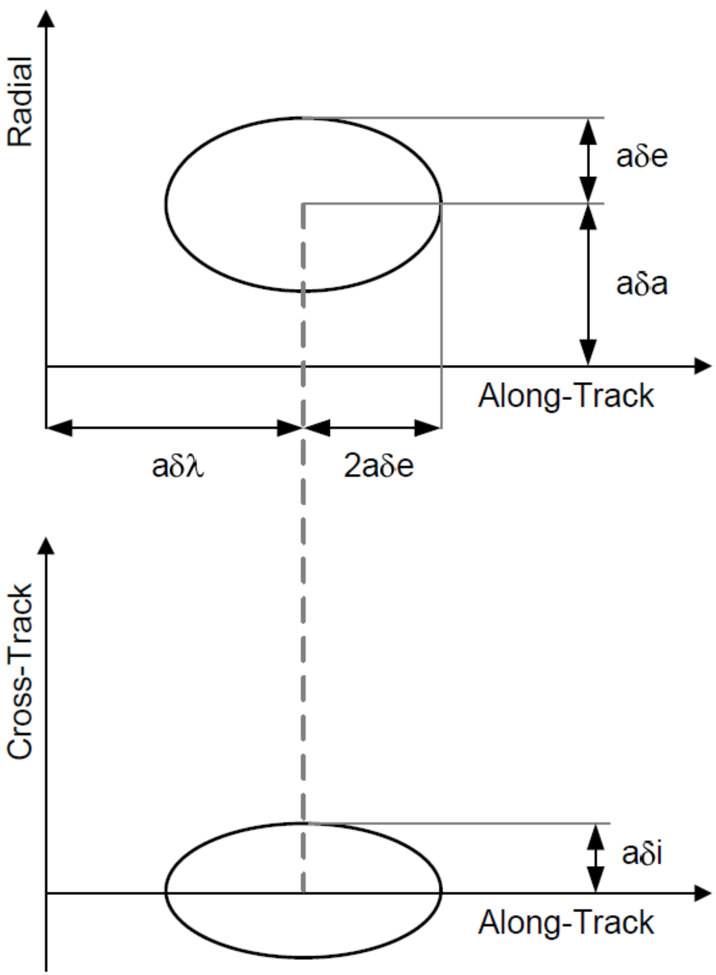
$$a\delta\lambda(t) = \frac{3}{4n^2}\Delta B \rho v^2 \left(u(t) - u_0\right)^2$$

$$a\delta a(t) = -\frac{1}{2n^2}\Delta B \rho v^2 \left(u(t) - u_0\right)$$



- Impact of differential drag can be minimized by employing identically designed spacecraft. The ballistic coefficients can be matched to roughly 1% at launch.
- Mass variations during lifetime can cause an additional difference of 1%
- Considering typical atmospheric density values in LEO, differential accelerations of <10^1 nm/s^2 are encountered which require negligible delta-vs
- This conclusion is no longer valid during safe modes (10^2 nm/s^2) or for non-cooperative spacecraft where differential drag can match absolute drag

New State Transition Matrix based on ROE



Linearized motion + disturbances

$$\begin{pmatrix} \delta\dot{a} \\ \delta\boldsymbol{\alpha} \end{pmatrix}_t = \boldsymbol{\Phi}(t-t_0)\begin{pmatrix} \delta\dot{a} \\ \delta\boldsymbol{\alpha} \end{pmatrix}_{t_0}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ \Delta t & 1 & 0 & 0 & 0 & 0 & 0 \\ \frac{\nu}{2}\Delta t^2 & \nu\Delta t & 1 & 0 & 0 & \mu\Delta t & 0 \\ 0 & 0 & 0 & 1 & -\dot{\varphi}\Delta t & 0 & 0 \\ 0 & 0 & 0 & \dot{\varphi}\Delta t & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda\Delta t & 1 \end{bmatrix}$$

differential drag   mean J_2

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Maneuver Planning

- A relative orbit control system is necessary either to maintain the nominal formation geometry over the mission lifetime (i.e., formation keeping) or to acquire new formation geometries (i.e., formation reconfiguration)

- The inversion of the solution of the HCW equations expressed in terms of ROE provides the ideal framework to design closed-form deterministic impulsive maneuvering schemes

Current orbit location

$u \geq u_M$

Maneuver location

Desired variation of ROE ($\Delta\cdot$ missing)

$$
\begin{aligned}
a\delta a &\approx && +2\delta v_t/n && \\
a\delta\lambda &\approx -2\delta v_r/n && -3(u-u_{\mathrm{M}})\delta v_t/n && \\
a\delta e_x &\approx +\delta v_r \sin u_{\mathrm{M}}/n && +2\delta v_t \cos u_{\mathrm{M}}/n && \\
a\delta e_y &\approx -\delta v_r \cos u_{\mathrm{M}}/n && +2\delta v_t \sin u_{\mathrm{M}}/n && \\
a\delta i_x &\approx && && +\delta v_n \cos u_{\mathrm{M}}/n \\
a\delta i_y &\approx && && +\delta v_n \sin u_{\mathrm{M}}/n
\end{aligned}
$$

Instantaneous variation of velocity

Maneuver Planning: Out-of-Plane

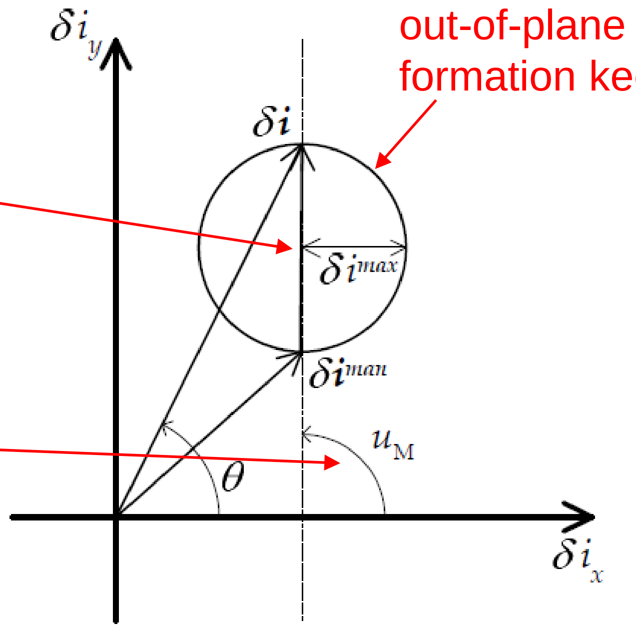
- The problem consists of 2 unknowns δv_n, u_M and 2 equations and can be solved through a single- or double-impulse

Magnitude of desired correction vector

$$
\begin{aligned}
\delta v_n &= na \left\| \delta \boldsymbol{i}^{\mathrm{man}} - \delta \boldsymbol{i} \right\| \\
u_{\mathrm{M}} &= \arctan \left[(\delta i_y^{\mathrm{man}} - \delta i_y) / (\delta i_x^{\mathrm{man}} - \delta i_x) \right]
\end{aligned}
$$

Phase of desired correction vector

Control window for out-of-plane formation keeping



Maneuver Planning: In-Plane (1)

- The problem consists of 3 unknowns δv_r, δv_t, u_M and 4 equations (overdetermined) and can be solved exactly only through a double-impulse scheme which doubles the number of unknowns (underdetermined)

$$
\begin{aligned}
\delta v_{t_1} &= \frac{na}{4}\left[\delta a + \delta e\cos(u_{\mathrm{M}_1}-\xi)\right] && -\frac{na}{4}\chi\left[\frac{\delta\lambda}{2} + \delta e\sin(u_{\mathrm{M}_1}-\xi)\right]\\
\delta v_{r_1} &= \frac{na}{2}\left[-\frac{\delta\lambda}{2} + \delta e\sin(u_{\mathrm{M}_1}-\xi)\right] && -\frac{na}{2}\chi\left[\delta a - \delta e\cos(u_{\mathrm{M}_1}-\xi)\right]
\end{aligned}
$$

$$
\begin{aligned}
\delta v_{t_2} &= \frac{na}{4}\left[\delta a - \delta e\cos(u_{\mathrm{M}_1}-\xi)\right] && +\frac{na}{4}\chi\left[\frac{\delta\lambda}{2} + \delta e\sin(u_{\mathrm{M}_1}-\xi)\right]\\
\delta v_{r_2} &= \frac{na}{2}\left[-\frac{\delta\lambda}{2} - \delta e\sin(u_{\mathrm{M}_1}-\xi)\right] && +\frac{na}{2}\chi\left[\delta a - \delta e\cos(u_{\mathrm{M}_1}-\xi)\right]
\end{aligned}
$$

First maneuver location

Second maneuver location

$$\chi = \frac{\sin(\Delta u_{\mathrm{M}})}{\cos(\Delta u_{\mathrm{M}}) - 1}$$

$$\xi = \arctan(\delta e_y/\delta e_x)$$

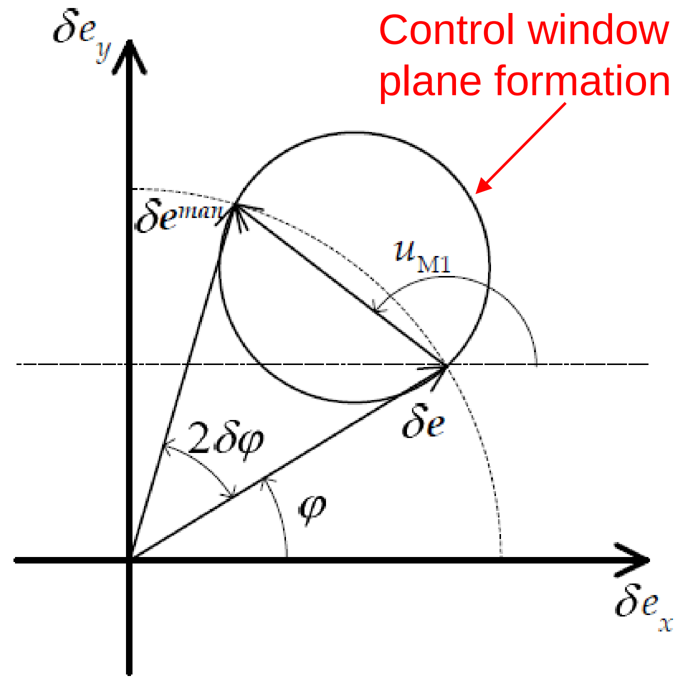
$$u_{\mathrm{M}_2} - u_{\mathrm{M}_1} \in]0, 2\pi[$$

Maneuver Planning: In-Plane (2)

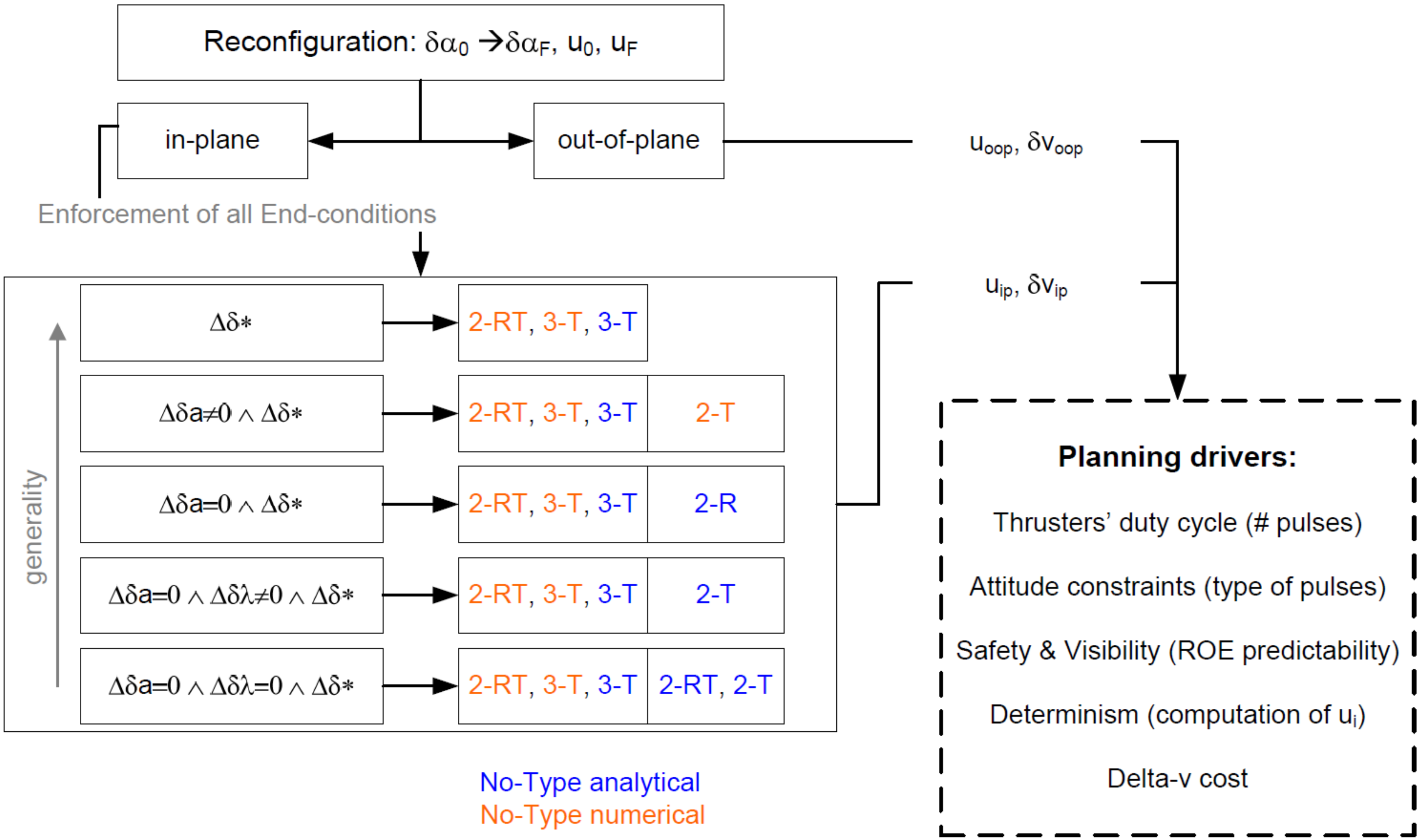
- The most simple double-impulse scheme with $u_{M1} = \xi$ and $u_{M2} = u_{M1} + \pi$ turns out to be the minimum cost (total delta-v) solution for formation keeping

$$
\begin{aligned}
\delta v_{t_1} &= \frac{na}{4}\left[(\delta a^{\mathrm{man}} - \delta a) + \|\delta \boldsymbol{e}^{\mathrm{man}} - \delta \boldsymbol{e}\|\right] \\
\delta v_{t_2} &= \frac{na}{4}\left[(\delta a^{\mathrm{man}} - \delta a) - \|\delta \boldsymbol{e}^{\mathrm{man}} - \delta \boldsymbol{e}\|\right] \\
u_{\mathrm{M}_1} &= \arctan\left[(\delta e_y^{\mathrm{man}} - \delta e_y)/(\delta e_x^{\mathrm{man}} - \delta e_x)\right]
\end{aligned}
$$

Note: $\delta\lambda$ is controlled through δa^{man} achieved after maneuver pair



Generalized Impulsive Maneuvering



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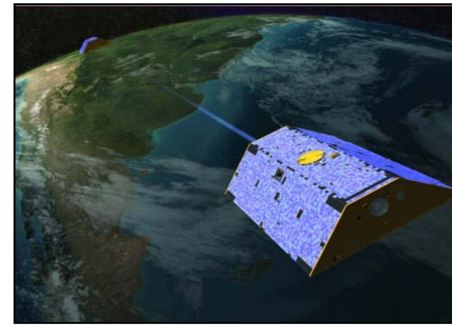
Advantages of Relative Orbital Elements

- Insight into geometry of relative motion and its relation to absolute motion
- Simple design of passively safe and stable relative orbits
- Straightforward introduction of perturbations into Hill's solution
- Simple interpretation of the effects of delta-v maneuvers
- Simple relationships between control windows and maneuver budget

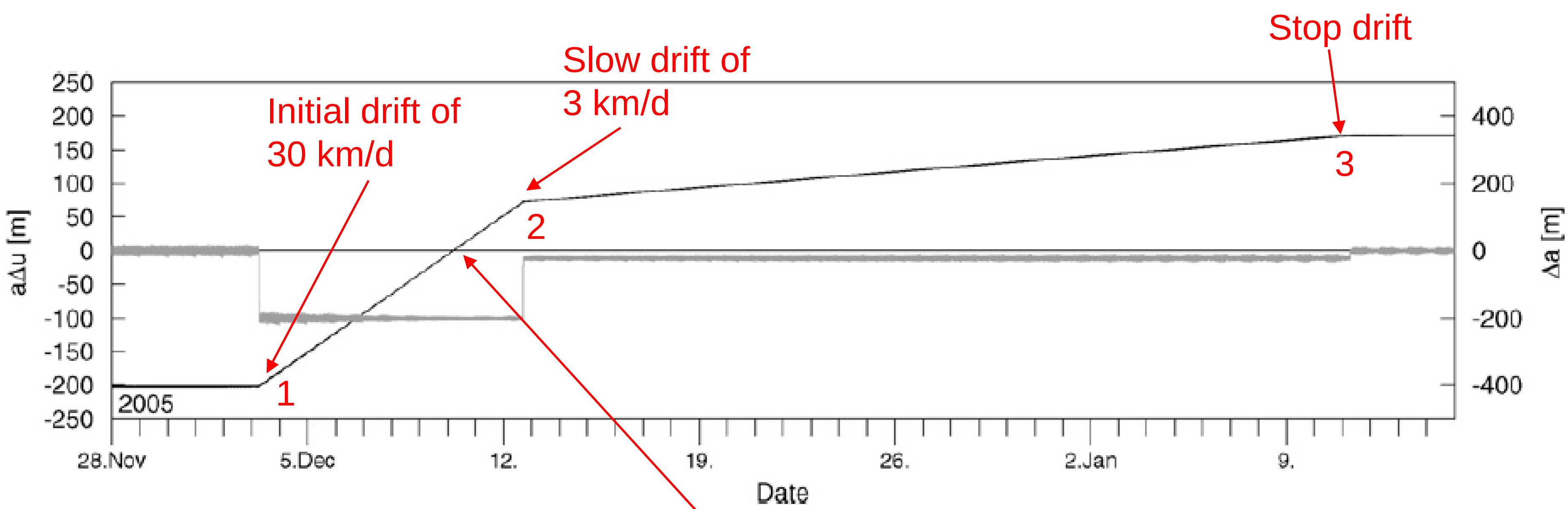
- This theory is used in a variety of formation-flying and rendezvous missions either operational or under development

GRACE – The Longitude Swap (1)



- After more than 2 years in orbit, a longitude swap maneuver was required to exchange the leading and trailing spacecraft of the GRACE formation
- While the two satellites are nominally separated by about 220 km in along-track direction, a close encounter took place during the swap sequence
- Taking care of the natural evolution of the relative orbital elements of GRACE 1 and 2, optimum maneuver dates were identified
- The fuel optimal maneuver sequence guaranteed a minimum distance during the encounter even in case of arbitrary thruster performance errors

GRACE – The Longitude Swap (2)



Maneuver	Date	UTC	δt	$a \cdot \Delta u$	u	δv_{plan}	δv	ε
1	2005/12/03	06:06	688 s	−200 km	298°	−10.89 cm/s	−11.19 cm/s	2.8%
2	2005/12/12	17:10	611 s	+70 km	263°	+9.82 cm/s	+9.93 cm/s	1.1%
3	2006/01/11	12:53	54 s	+170 km	146°	+0.86 cm/s	0.86 cm/s	0.0%

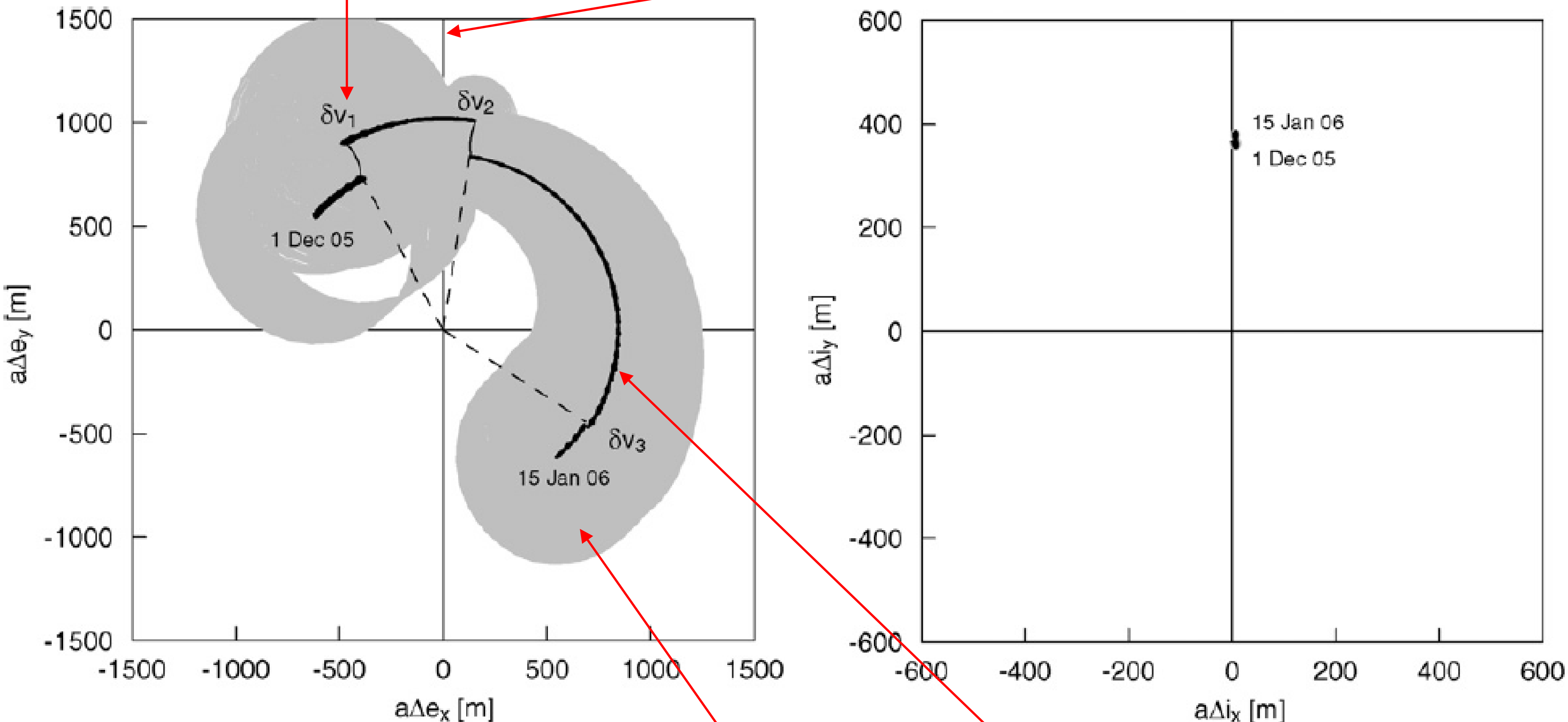
Problem: 5% maneuver execution error causes an uncertainty of 8h over 7 days

Fact: concise forecast of relative motion near encounter is impossible

GRACE – The Longitude Swap (3)

delta-v does not change relative perigee

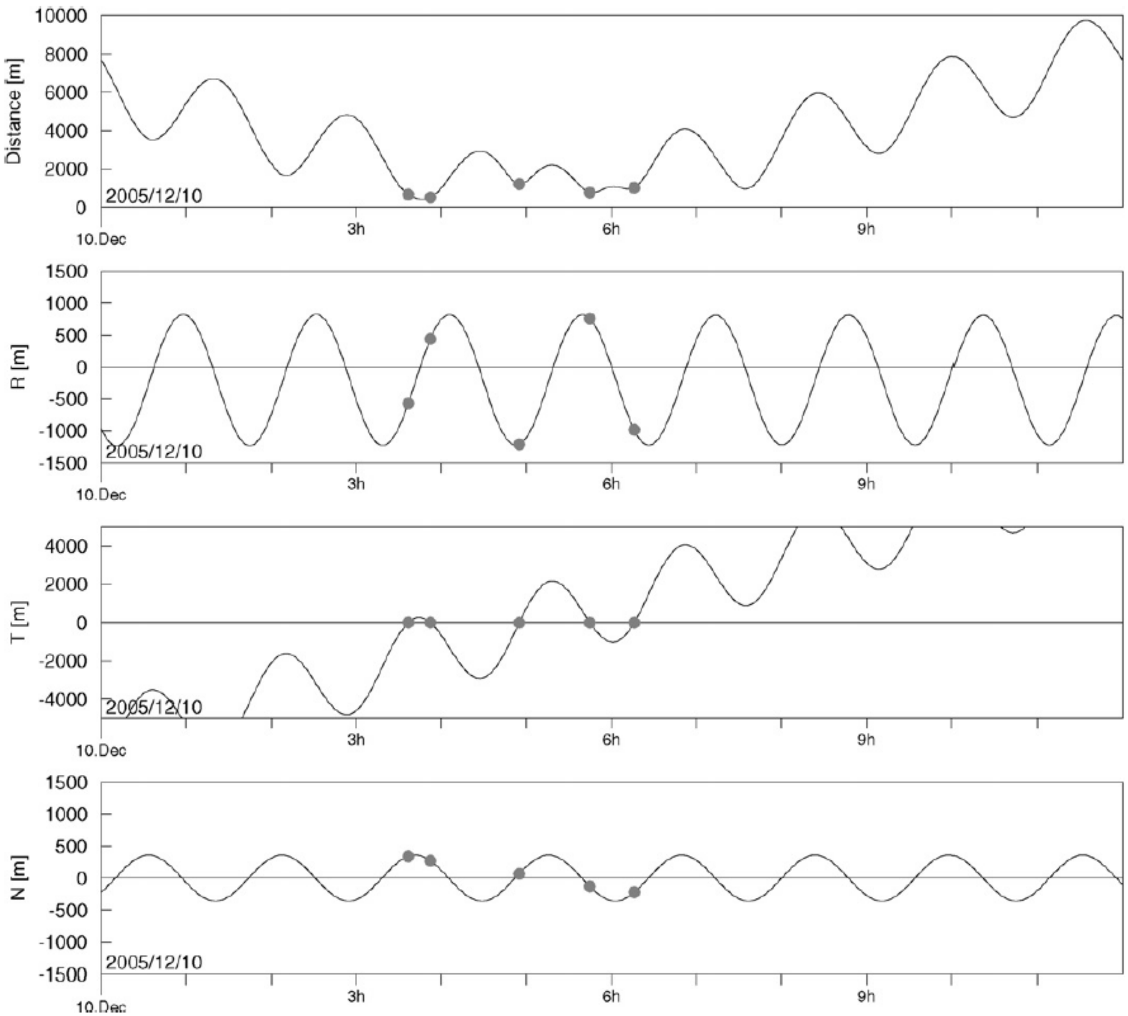
Parallel configuration naturally achieved on Dec. 10, 2005

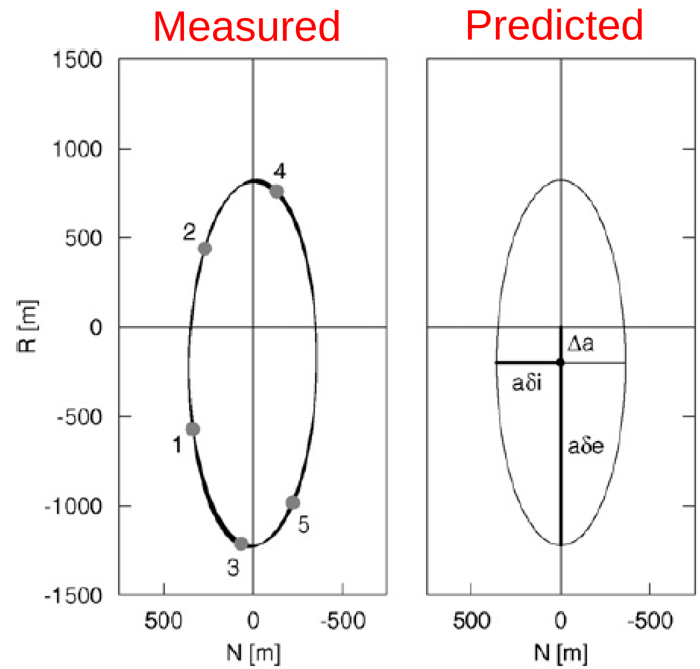


Osculating relative eccentricity vector

Mean relative eccentricity vector

GRACE – The Longitude Swap (4)

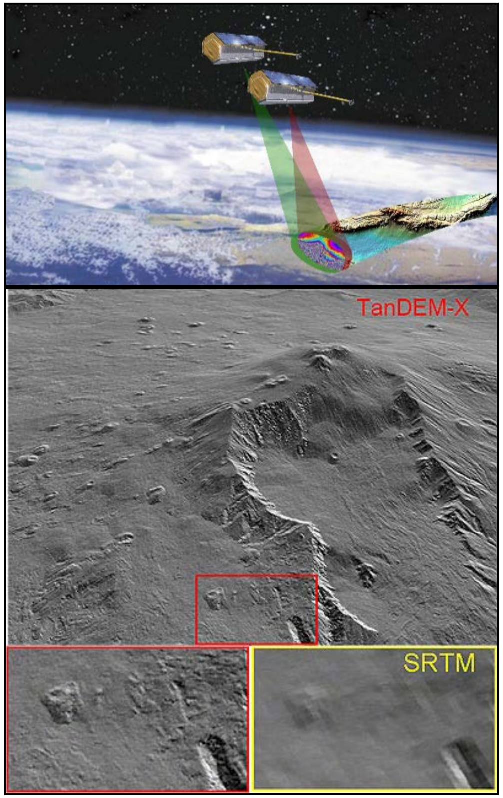




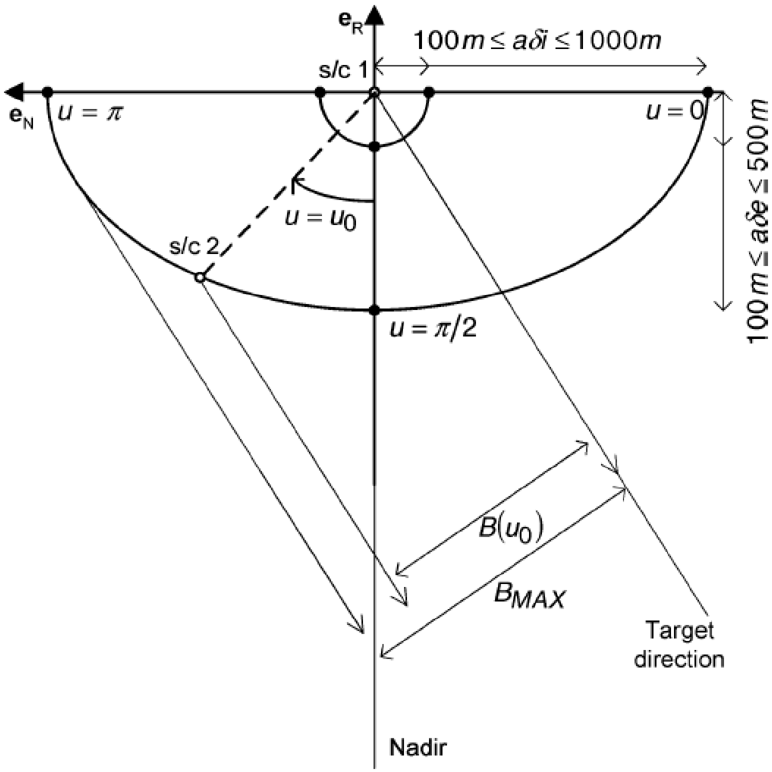
Parallel configuration gave a 90° phase shift of the periodic radial and cross-track motion which ensured a safe minimum separation >431 m at all times

TanDEM-X – The Formation Design (1)

- TanDEM-X represents the first operational formation-flying mission for Synthetic Aperture Radar (SAR) interferometry in low-Earth orbit
- Two nearly identical satellites, TSX and TDX, were launched with a two-year time shift in 2007 and 2009
- The mission profile is particularly challenging from a flight dynamics point of view and poses new needs for spacecraft navigation and control
- The relative orbital elements theory forms the basis of the formation design, navigation and control concept



TanDEM-X – The Formation Design (2)



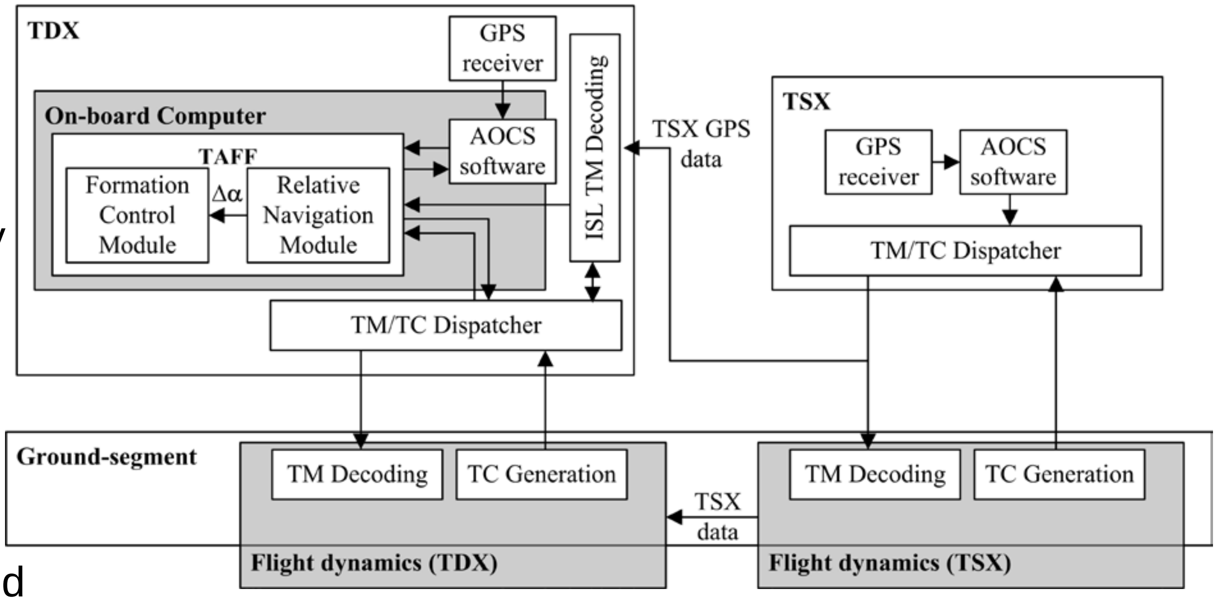
Mission phase	$a\delta e$ [m]	$a\delta i$ [m]	$\theta - \varphi$	Days of mission	Notes
C1	260	222	200	103–179	Preliminary DEM generation with small baselines and large height of ambiguity
C2	297	260	210	180–300	(see above)
C3	297	371	210	301–399	(see above)
C4	350	350	180	399–465	High latitudes
D1	386	330	200	466–542	Final DEM generation with large baseline and small height of ambiguity
D2	440	385	210	543–663	(see above)
D3	440	550	210	664–762	(see above)
D4	500	500	180	763–828	High latitudes
E				829–839	Swap of relative eccentricity vector
F1	250	250	0	840–949	Crossing orbits, mountainous terrain
F2	250	600	0	950–982	(see above)
G1	250	1000	0	983–1015	Separation of satellites in right ascension of ascending node
G2	250	2000	0	1016–1048	(see above)
G3	250	4000	0	1049–1081	(see above)

$$|\Delta v| = \frac{1}{2} v \left| \frac{\mathrm{d}\Delta e}{\mathrm{d}t} \cdot 1\mathrm{d} \right| \approx 3 \times 10^{-5} (a\delta e)/\mathrm{s}$$

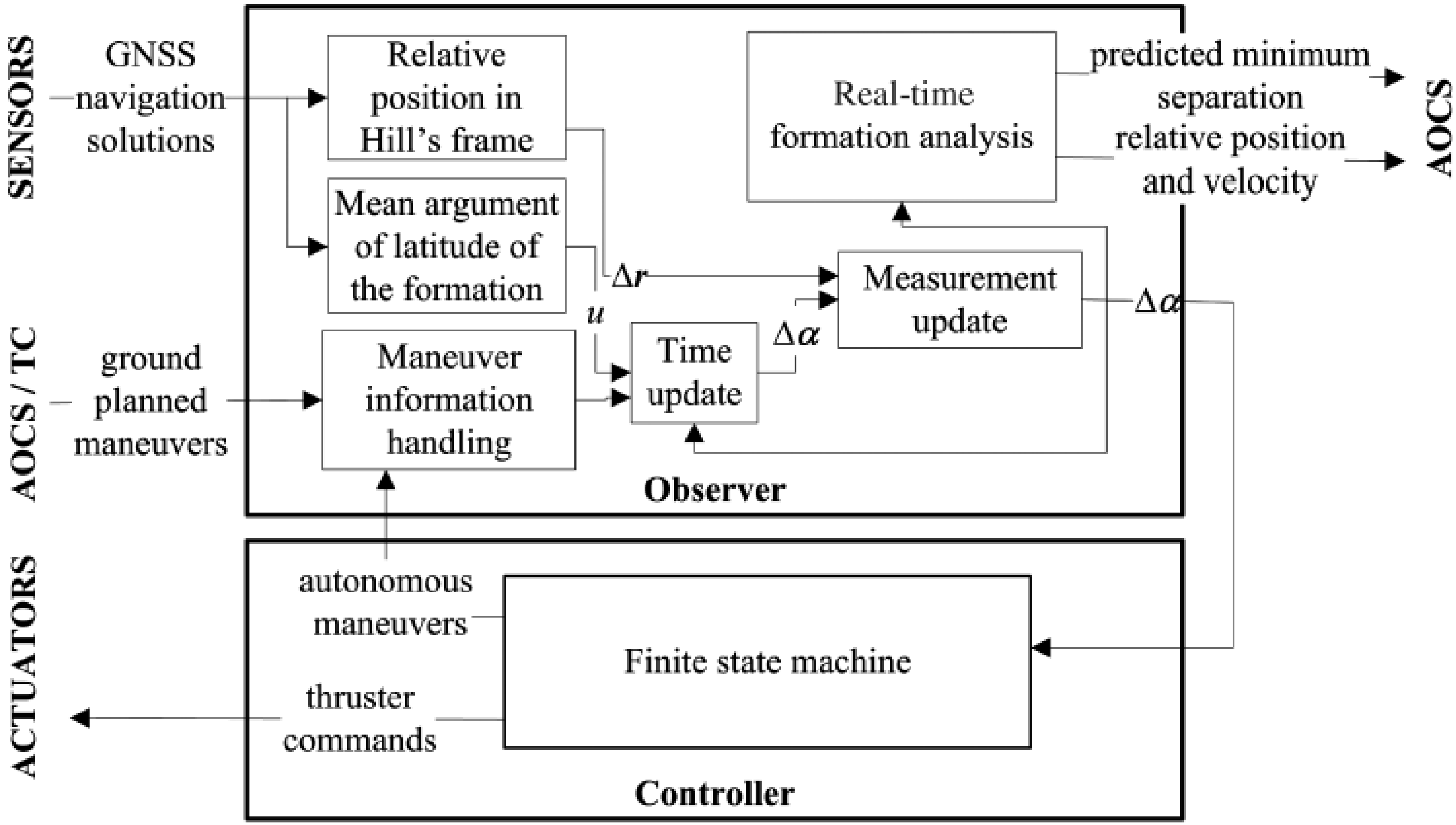
$a\delta e$ = 300m requires 2 burns/day of 0.5 cm/s each and separated by π

TanDEM-X – The TAFF System (1)

- Higher control accuracy enables SAR applications
- Autonomy implies simplicity of mission operations
- Design drivers are simplicity and robustness (KISS)
- Kalman filter for relative navigation
- GPS navigation solutions adopted as measurements
- Navigation and control based on relative orbital elements



TanDEM-X – The TAFF System (2)



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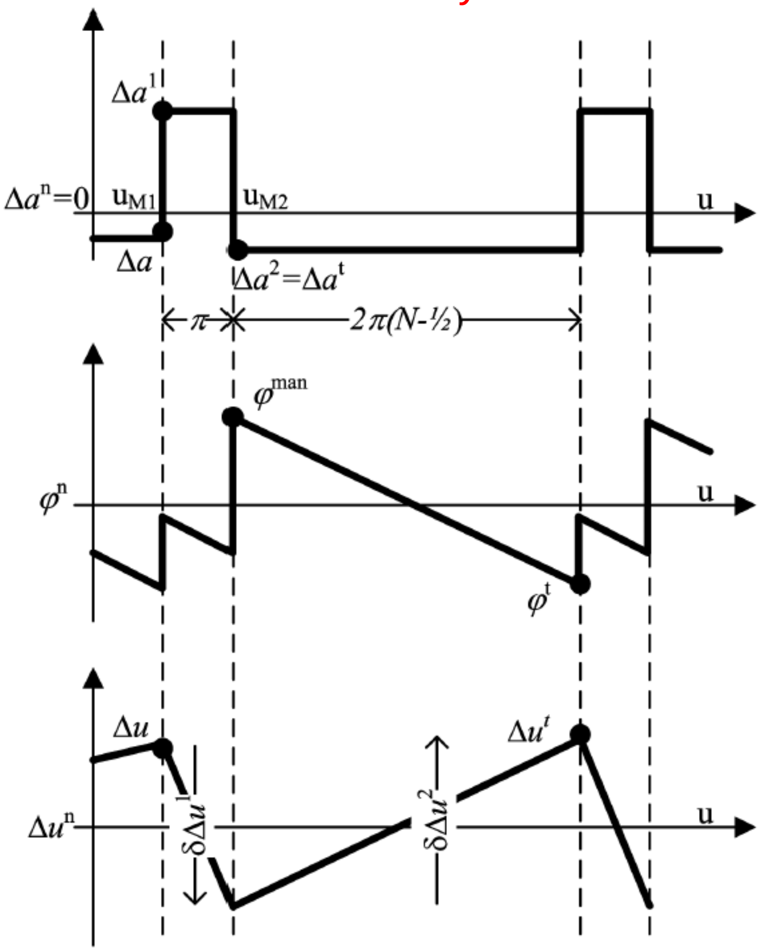
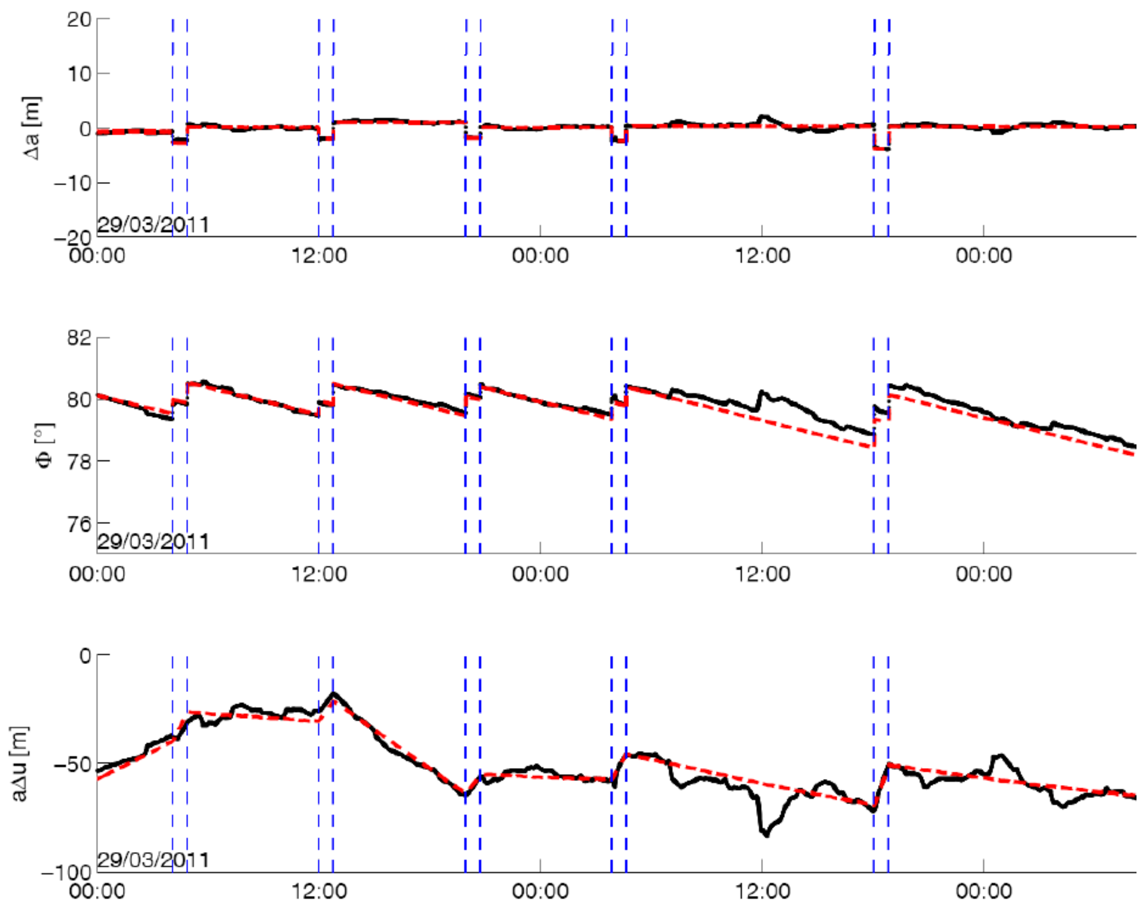
TanDEM-X – The TAFF System (3)

Theory



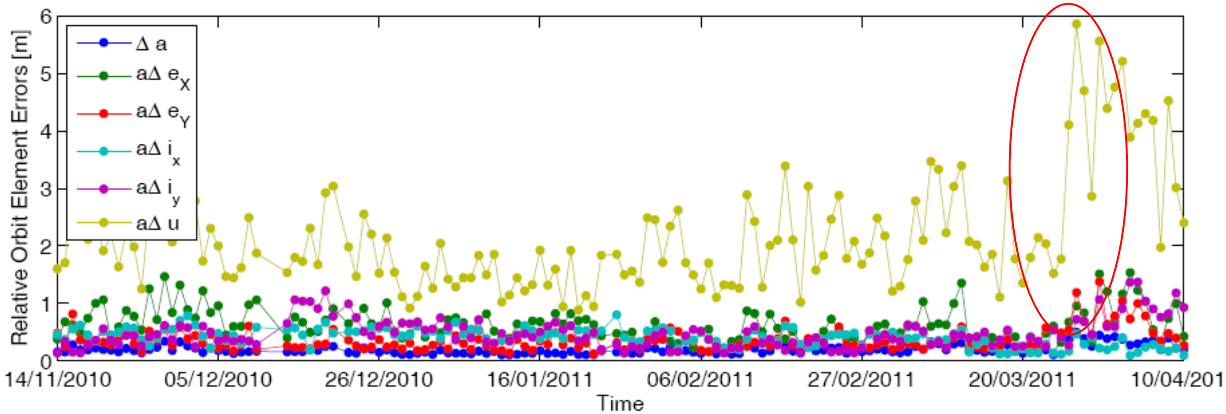
Early Flight Results

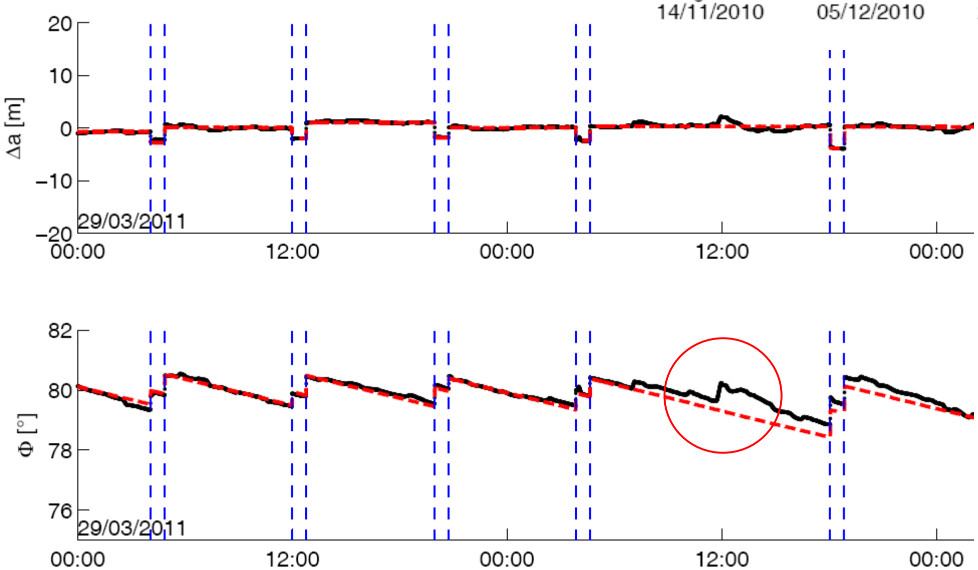


TanDEM-X – The TAFF System (4)

Mar. 2011 (Jul. 2012)

- Relative orbital elements accuracy at m level (rms)
- Degraded accuracy during high solar activity

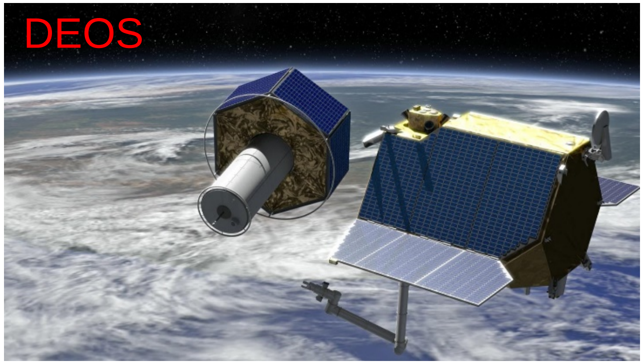




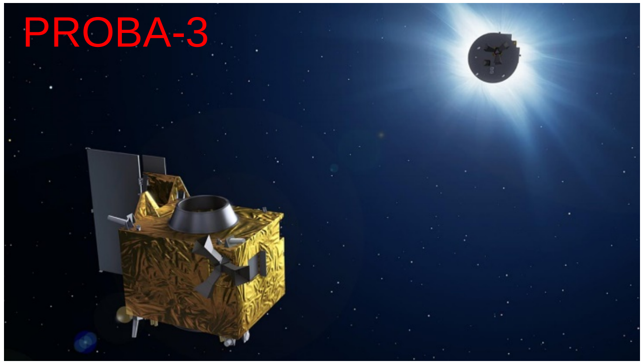
- In-plane formation keeping
  - First campaign: March 29-31, 2011
  - Goal: TAFF commissioning
- Early flight results
  - Higher accuracy than ground-based
  - Control errors < 7.5/36.3 m [R/T]
  - Robustness to navigation outliers

Outlook

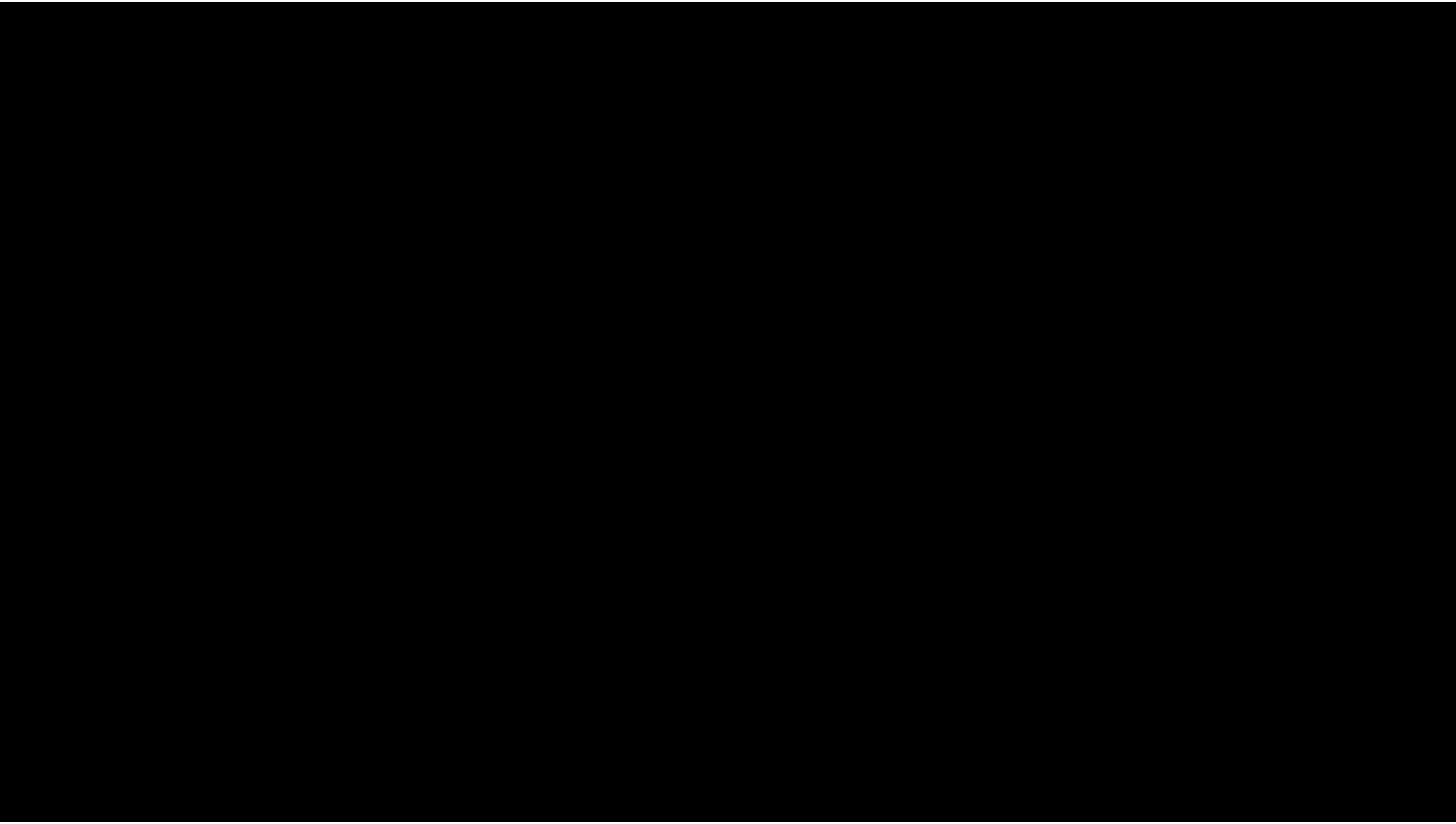
- Far- to mid-range rendezvous to a non-cooperative target
  - Angles-only navigation
  - Differential drag
  - Active collision avoidance



- Formation acquisition and break-up in high elliptical orbits
  - Passive and active safety
  - Maneuver planning



DEOS (Mission Objectives and Scenario)



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References

1. D'Amico S.; *Autonomous Formation Flying in Low Earth Orbit*; PhD thesis; Technical University of Delft, 2010.
2. Schaub H.; *Relative Orbit Geometry through Classical Orbit Element Differences*; Journal of Guidance, Control and Dynamics, 27(5),Sept–Oct 2004.
3. D'Amico S., Montenbruck O.; *Proximity Operations of Formation Flying Spacecraft using an Eccentricity/Inclination Vector Separation*; Journal of Guidance, Control and Dynamics, 29/3 554-563, 2006.
4. D'Amico S.; *Relative Orbital Elements as Integration Constants of the Hill's Equations*; DLR-GSOC TN 05-08; Deutsches Zentrum für Luft- und Raumfahrt, Oberpfaffenhofen, 2005.
5. Montenbruck O., Kirschner M., D'Amico S., Bettadpur S.; *E/I-Vector Separation for Safe Switching of the GRACE Formation*; Aerospace Science and Technology 10/7, 628-635, 2006.
6. Ardaens J.-S., D'Amico S.; *Spaceborne Autonomous Relative Control System for Dual Satellite Formations*; Journal of Guidance, Control and Dynamics 32(6):1859-1870, 2009.
7. Gaias G., D'Amico S.; Impulsive Maneuvers for Formation Reconfiguration using Relative Orbital Elements; Journal of Guidance, Control, and Dynamics (2014). In print. DOI: 10.2514/1.G000189

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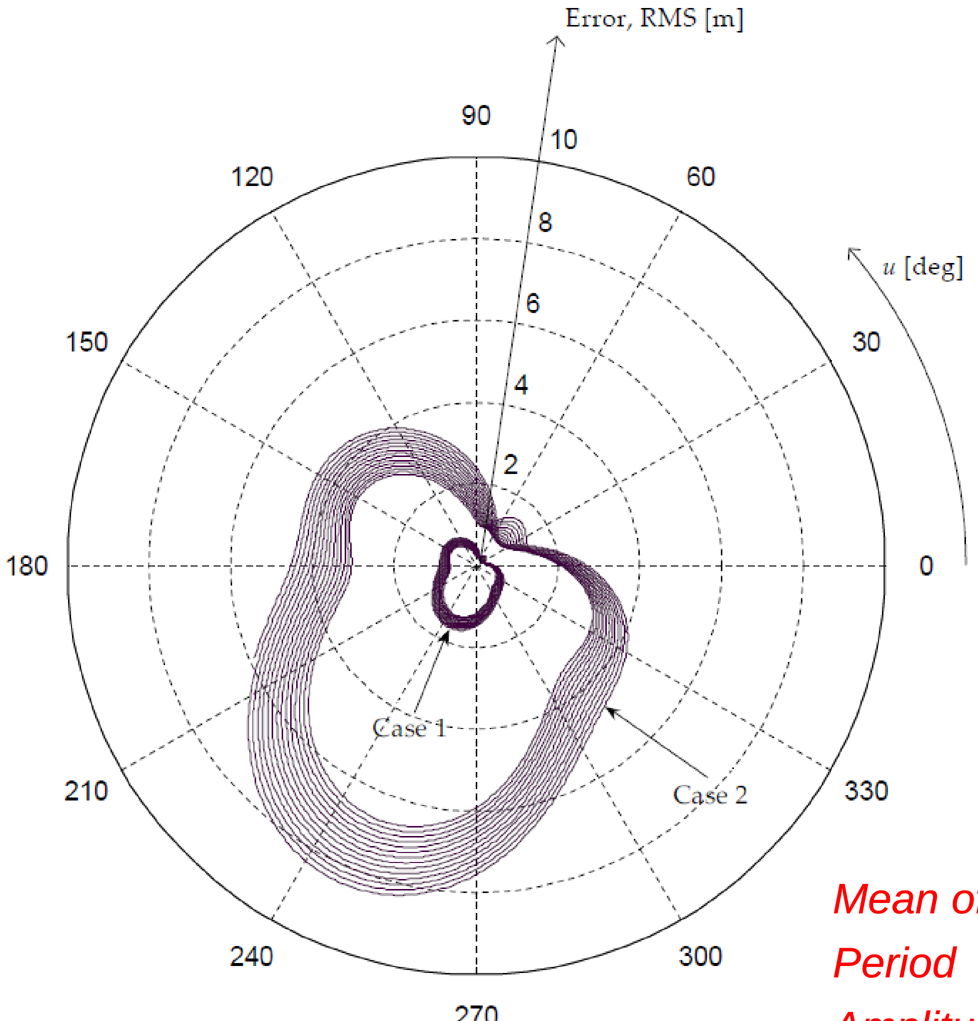
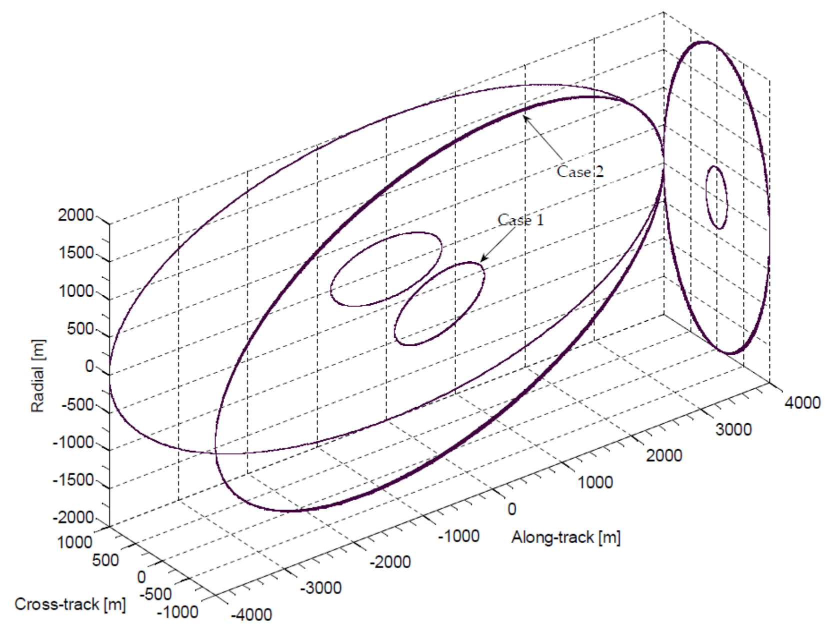
Backup

Accuracy of Linear ROE-Based Model



Orbit elements	Value	Relative orbit elements	Value
a [m]	7078135.0	$a\delta a$ [m]	0
u [°]	0.0	$a\delta\lambda$ [m]	0
e_x [-]	0.001	$a\delta e_x$ [m]	0
e_y [-]	0.0	$a\delta e_y$ [m]	400^1 or 2000^2
i [°]	98.19	$a\delta i_x$ [m]	0
Ω [°]	189.89086	$a\delta i_y$ [m]	200^1 or 1000^2



Mean offset – Due to initialization (mean to osc)

Period – Orbital period

Amplitude – About baseline/100 (< 15 km)

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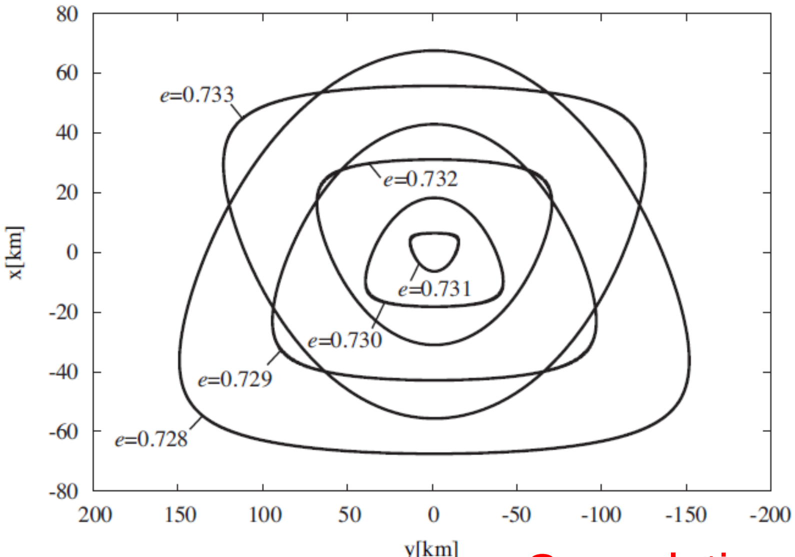
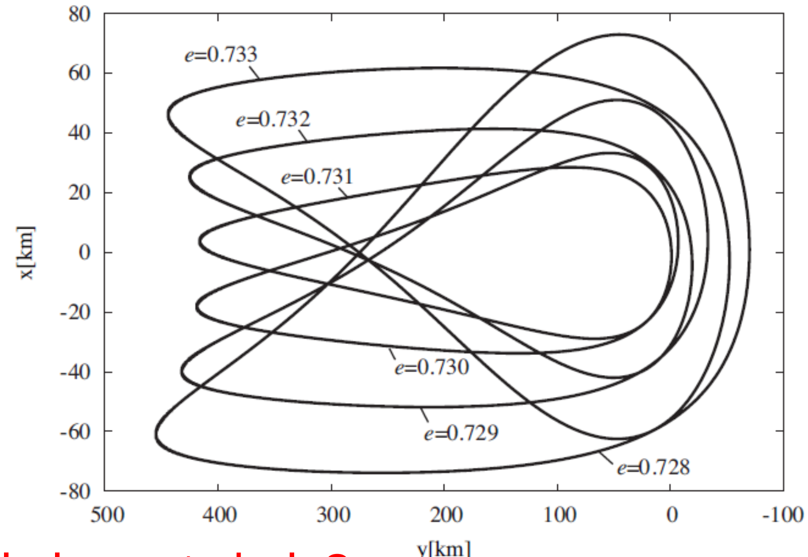
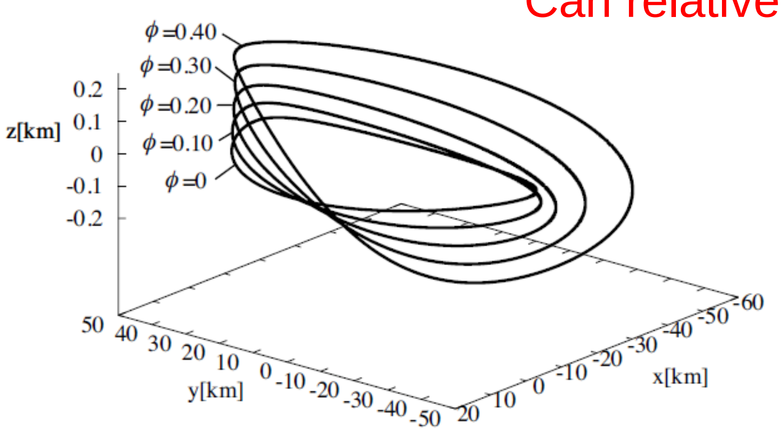
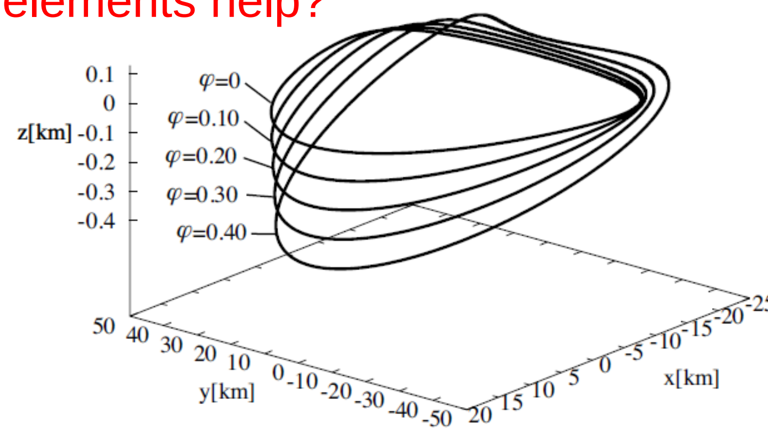
Relative Motion in Eccentric Orbits





Can relative orbital elements help?





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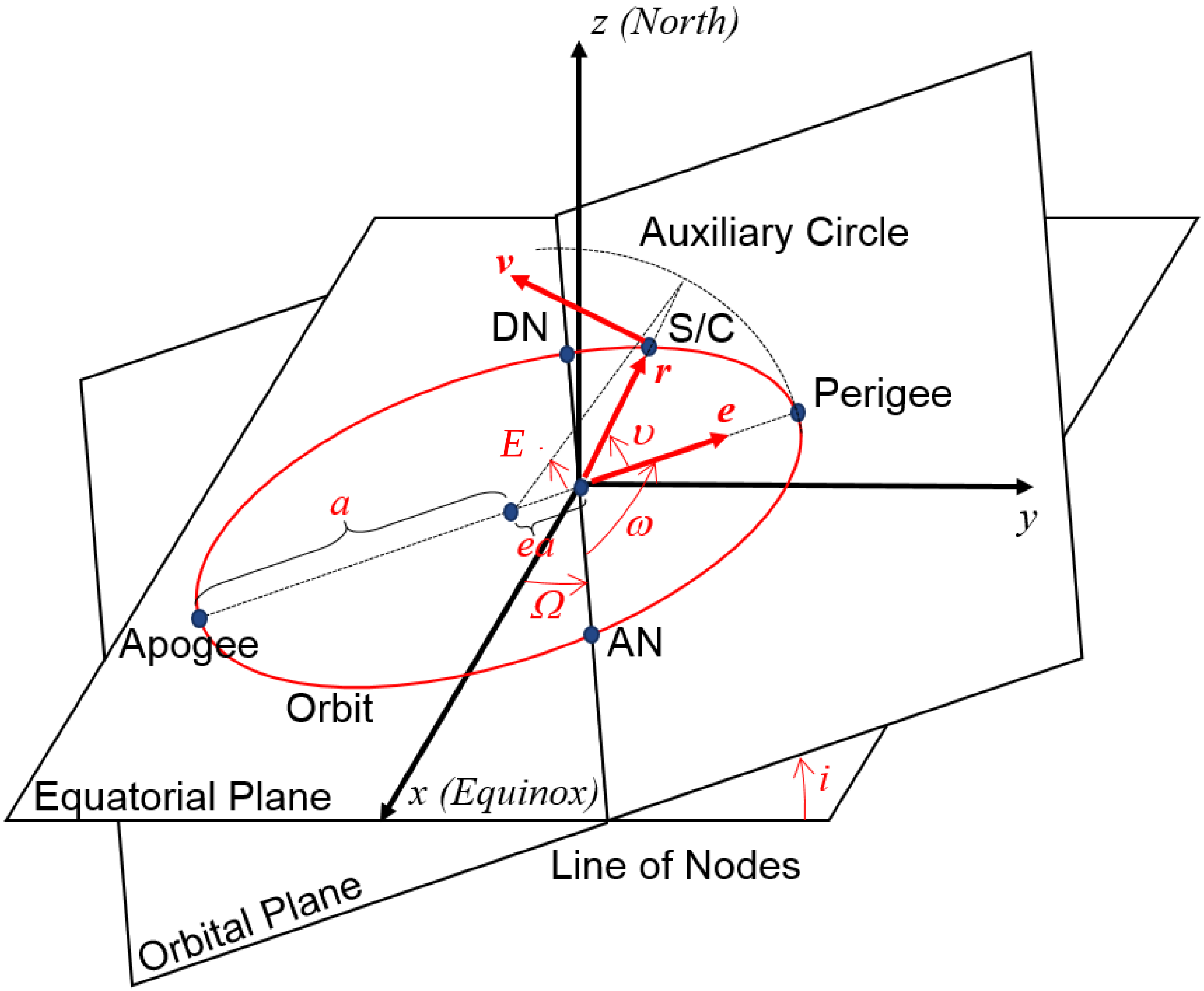
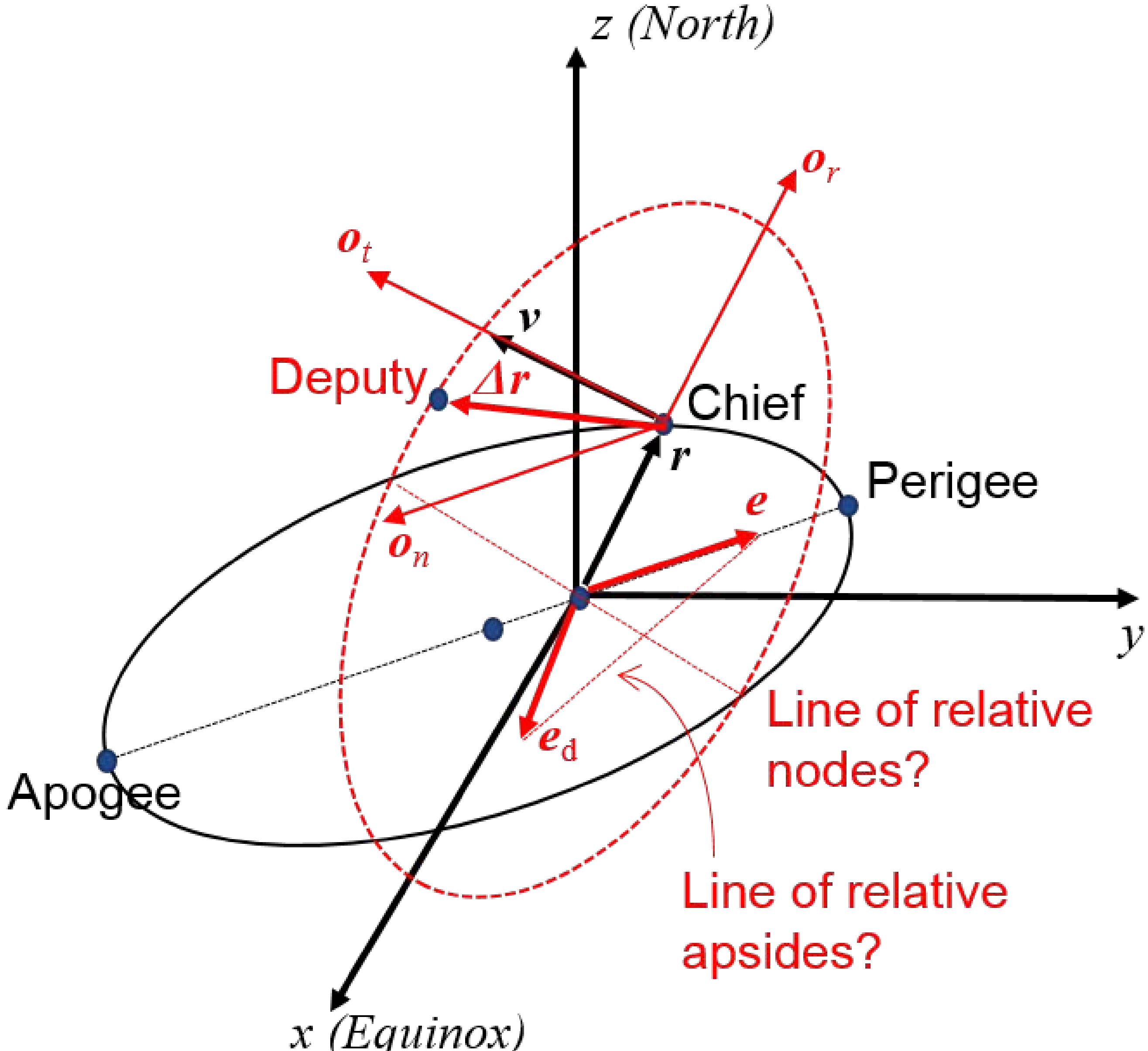

z (North)
Auxiliary Circle
v
DN
S/C
r
Perigee
e
E
υ
a
ea
ω
y
Ω
Apogee
AN
Orbit
Equatorial Plane
x (Equinox)
i
Line of Nodes
Orbital Plane


z (North)
o_r
o_t
v
Deputy
Δr
Chief
r
Perigee
e
o_n
y
e_d
Line of relative nodes?
Apogee
Line of relative apsides?
x (Equinox)